

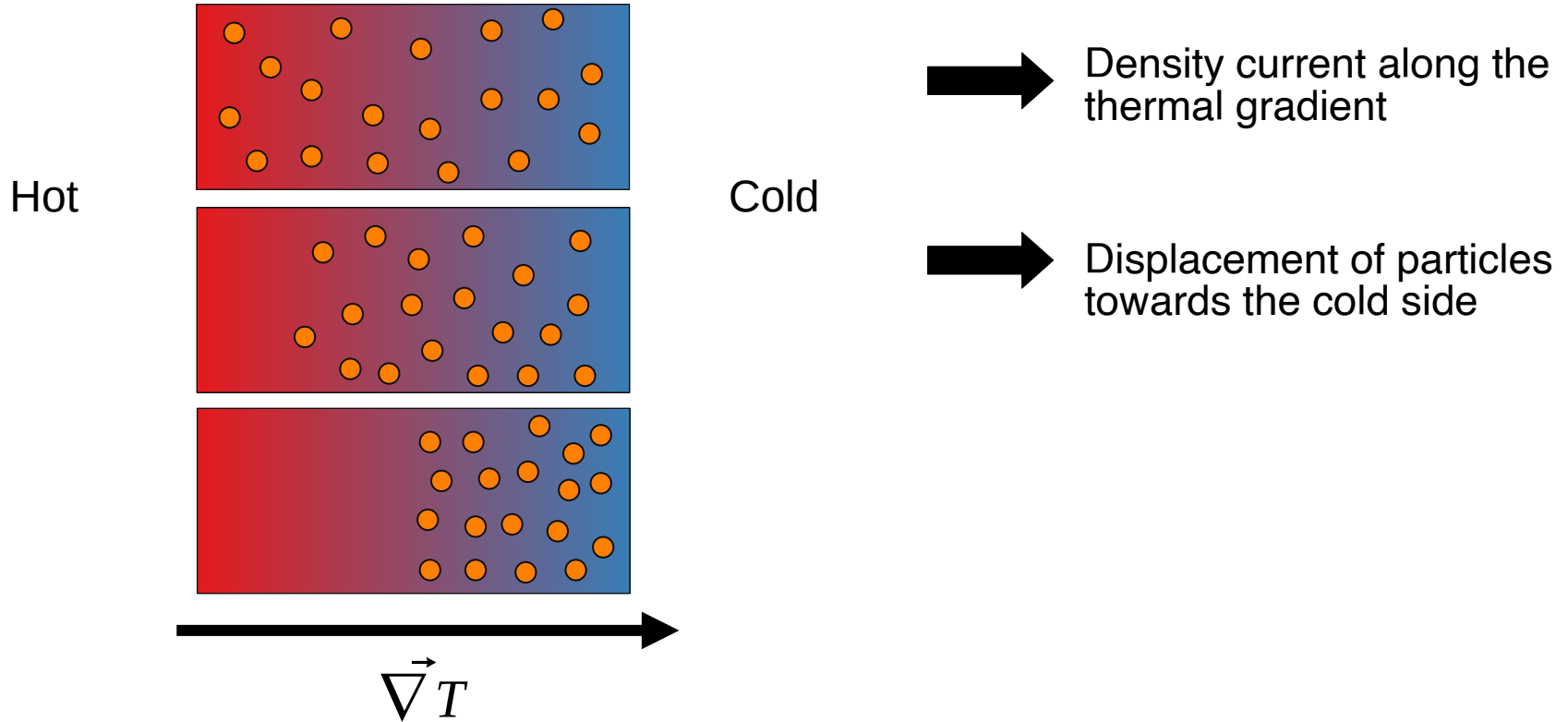
THERMAL MASS TRANSPORT MECHANISM OF AN ADATOM ON A CRYSTALLINE SURFACE

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THERMOMIGRATION: DEFINITION

Thermomigration: diffusion of particles subjected to a thermal gradient



THERMOMIGRATION: BACKGROUND

Liquids —————> Ludwig(1856) and Soret(1879) ✓

Gases —————> Tyndall(1870) and Strutt(1882) ✓

Solids —————> Pfann (1955) and Huttington (1968) ✓

Surfaces —————> Less studied ✗

Recent experiments (growth [1, 2], transport [3,4]) involving surface thermomigration

- [1] Xie, et al. Controlled growth of single-crystalline metal nanowires via thermomigration across a nanoscale junction. Nat Commun 10, 4478 (2019).
- [2] Samarao, et al. Temperature Compensation of Silicon Resonators via Degenerate Doping. IEEE Transactions on Electron Devices. 59. 1 – 4. (2010).
- [3] Barreiro, et al. Subnanometer motion of cargoes driven by thermal gradients along carbon nanotubes. Science. 320, 5877 :775-8. (2008)
- [4] Leroy, et al. Determination of the Thermomigration Force on Adatoms. Phys. Rev. Lett.,131, 116202(2023)

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THERMOMIGRATION: TOOLS

Applied thermal gradients to surfaces:

➡ In materials, the transport of heat can be provided by :

➡ Propagation of electrons ➡ Metals

➡ Propagation of phonons ➡ Semiconductors and
insulators

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THERMOMIGRATION: TOOLS

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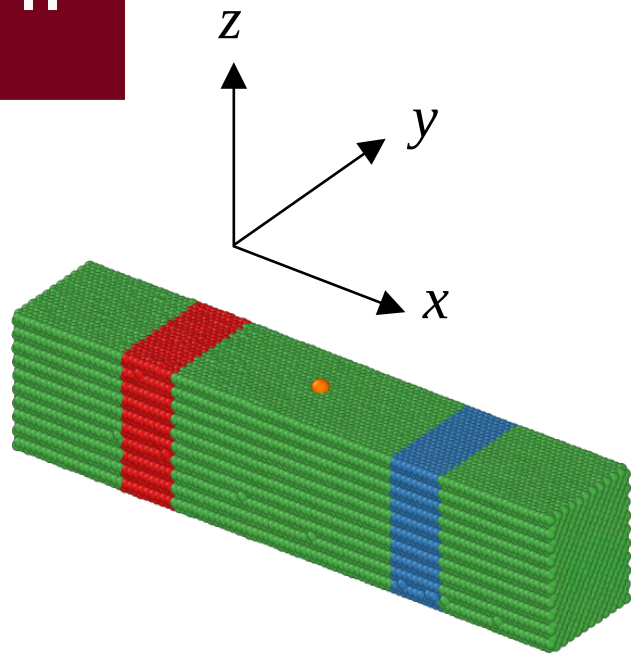
➔ In materials, the transport of heat can be provided by :

Objective: To understand the thermomigration of an atom or a cluster on a crystalline surface in the presence of a thermal gradient

Tools: theoretical study with molecular dynamics simulation

Insulators

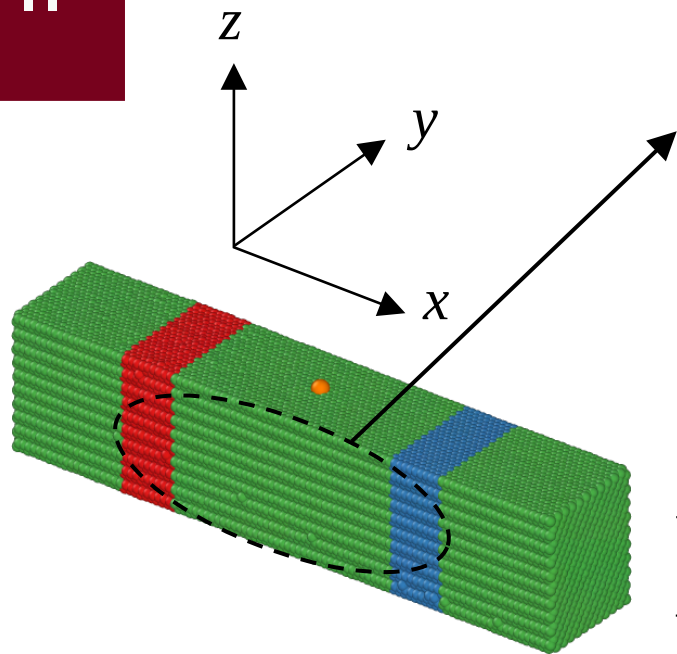
MODEL



Schematic of the simulation box

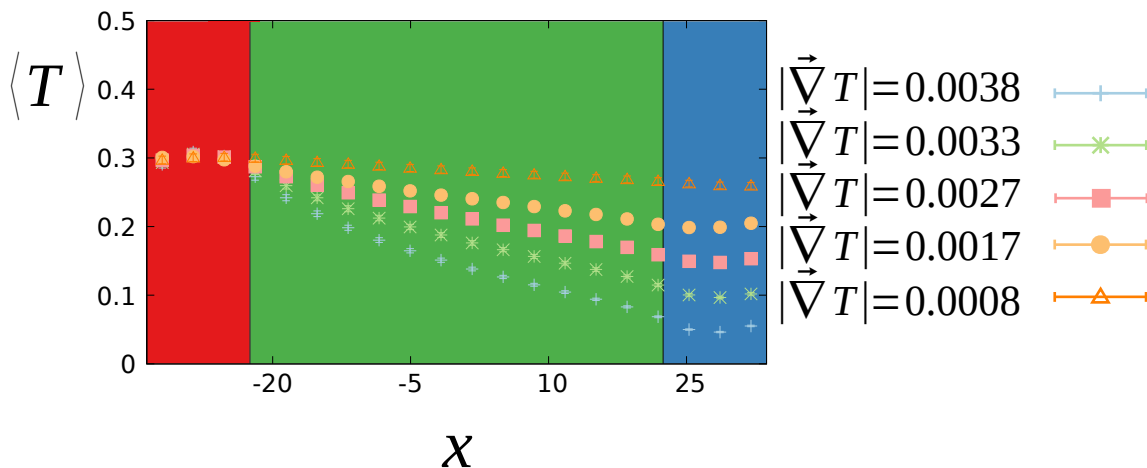
- Face-centered cubic crystal (FCC)
- Adatom on the (1,1,1) surface
- Atomic interactions → Lennard-Jones potential
→ reduced Lennard-Jones units
- Periodic conditions in x and y directions
- Hot and cold regions (Nosé-Hoover thermostats)

MODEL



Schematic of the simulation box

→ Substrate in the steady state



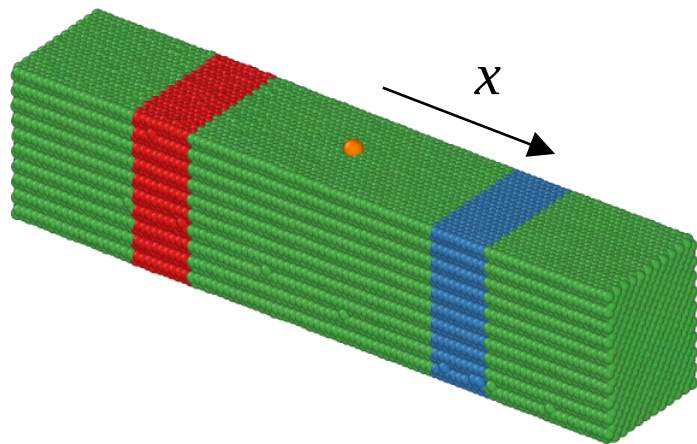
→ For the lowest gradients, the profile is linear.

→ For the highest gradients, the profile is non-linear.

→ Substrate leaves the domain of validity of Fourier's law

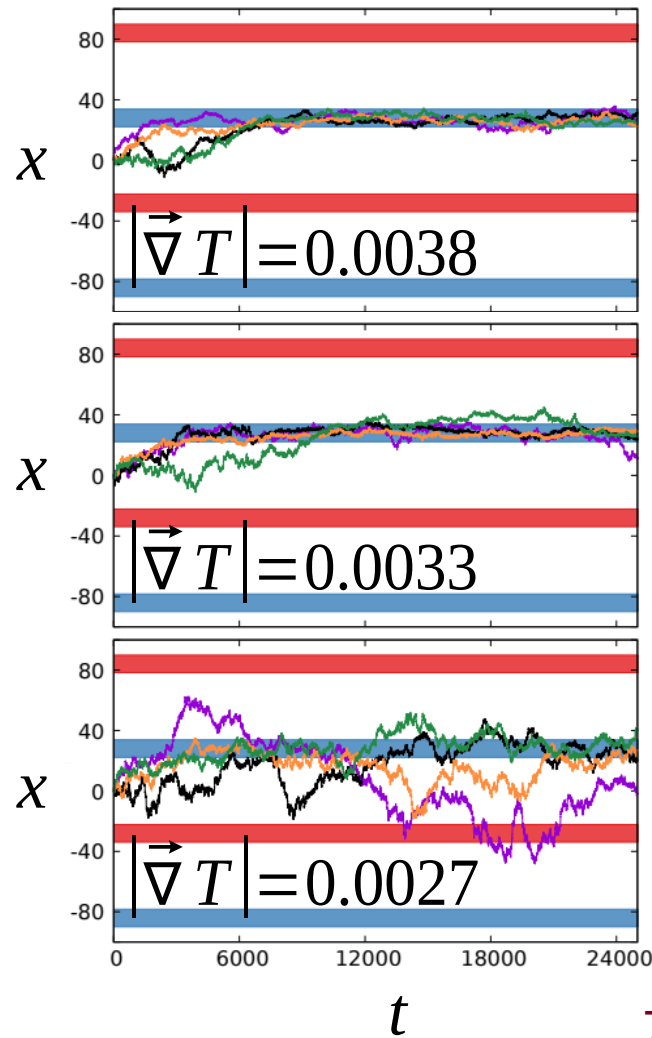


STUDY OF TRAJECTORIES



- ➡ The adatom moves towards the cold region
- ➡ Competition between the drift induced by the thermal gradient and the diffusion
- ➡ The weaker the gradient, the weaker the drift

We intend to decorrelate the drift induced by the thermal gradient and diffusion.



IV

THERMODYNAMIC POTENTIAL

hypothesis: the adatom is in local thermodynamic equilibrium

→ Probability of presence of the adatom is driven by : $p(x_0) \propto e^{-\frac{A(x_0)}{T(x_0)}}$

Thermodynamic potential $\Phi(x_0) = \frac{A(x_0)}{T(x_0)}$ → Not directly computable



$$\frac{\partial(\Phi(x_0))}{\partial x} = - \left\langle \frac{Fx_{adatom \leftrightarrow substrate}}{T(x_0)} \right\rangle - \left\langle \frac{\frac{Ep_{adatom \leftrightarrow substrate}(x_0, y, z)}{2} + Ec_{adatom}(vx, vy, vz)}{T(x_0)^2} \frac{\partial T(x_0)}{\partial x} \right\rangle$$

T Temperature
 Ep Potential energy
 Ek Kinetic energy
 F Force

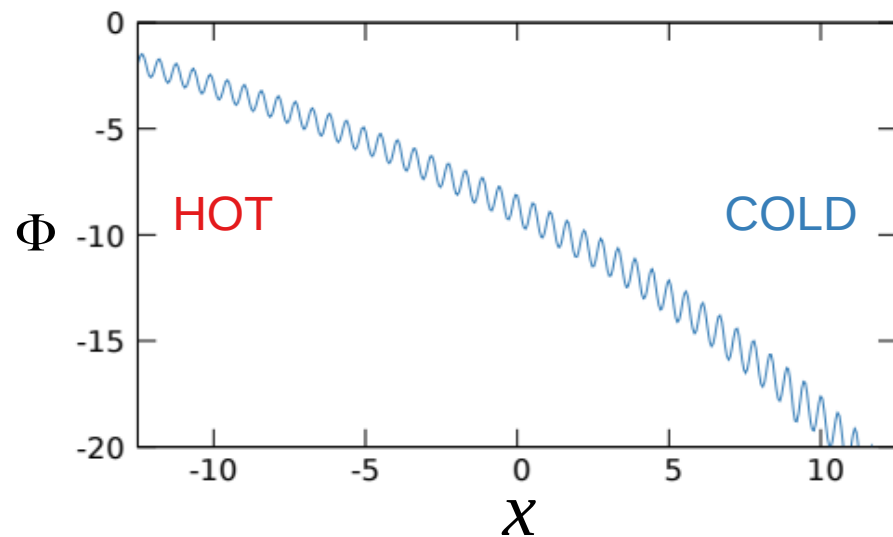
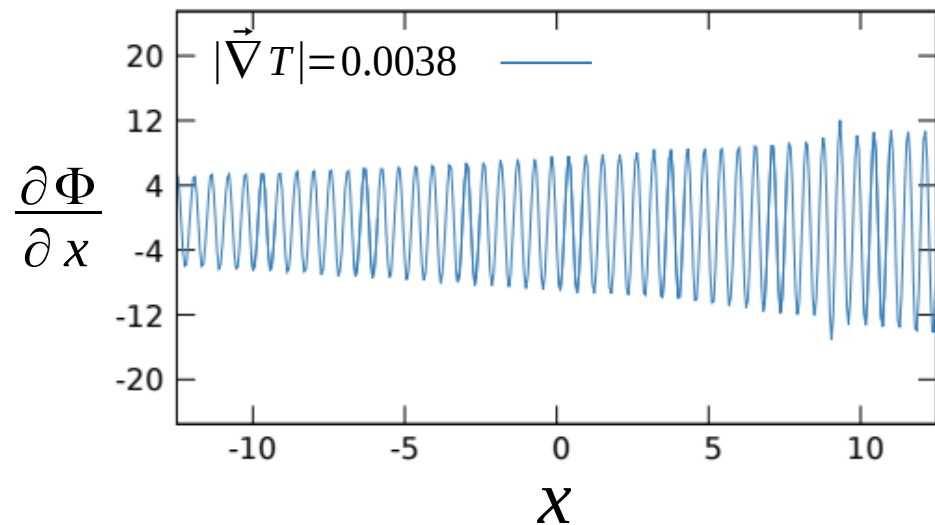


Generalization of thermodynamics integration for of a system subjected to a thermal gradient

IV

THERMODYNAMIC POTENTIAL

$$\Phi(x_0) = \frac{A(x_0)}{T(x_0)}$$



➡ Probability of presence is higher on the cold region than on the hot region $p(x_0) \propto e^{-\Phi(x_0)}$

IV

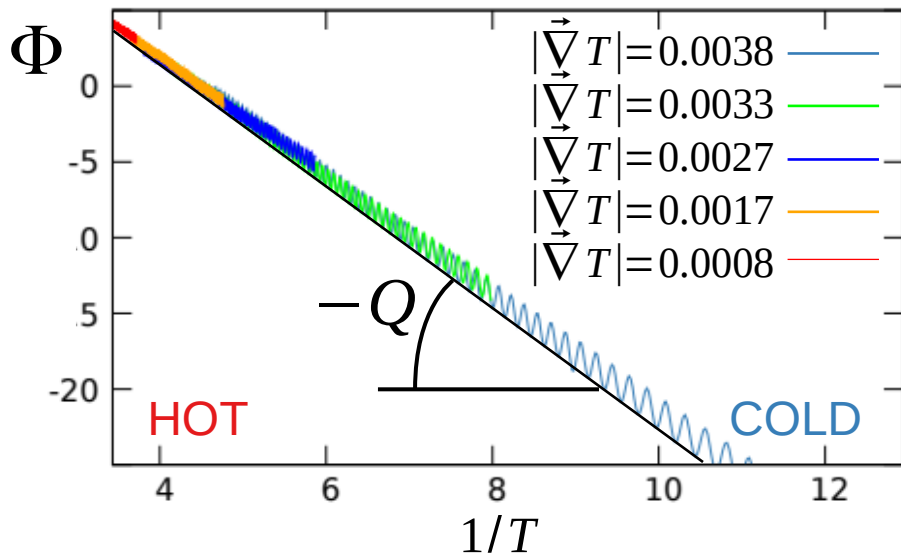
THERMODYNAMIC POTENTIAL

Φ depends on x
 T depends on x

Plot Φ as function of $\frac{1}{T}$

$$\Phi(x_0) = \frac{A(x_0)}{T(x_0)}$$

Characterize Φ by the slope Q which passes through all the minimums



Slope Q does not depend on the gradient

Slope Q related to the response of the system to a thermal gradient that drives the adatom towards the cold region

$Q \rightarrow$ drift

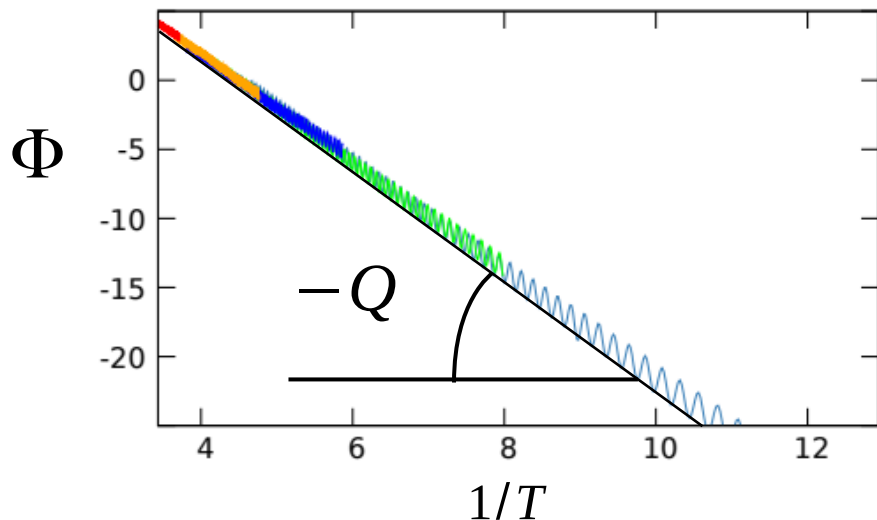
From the expression of $\frac{\partial \Phi}{\partial x}$, Q is related to the binding energy

IV

THERMODYNAMIC POTENTIAL

$$\Phi(x_0) = \frac{A(x_0)}{T(x_0)}$$

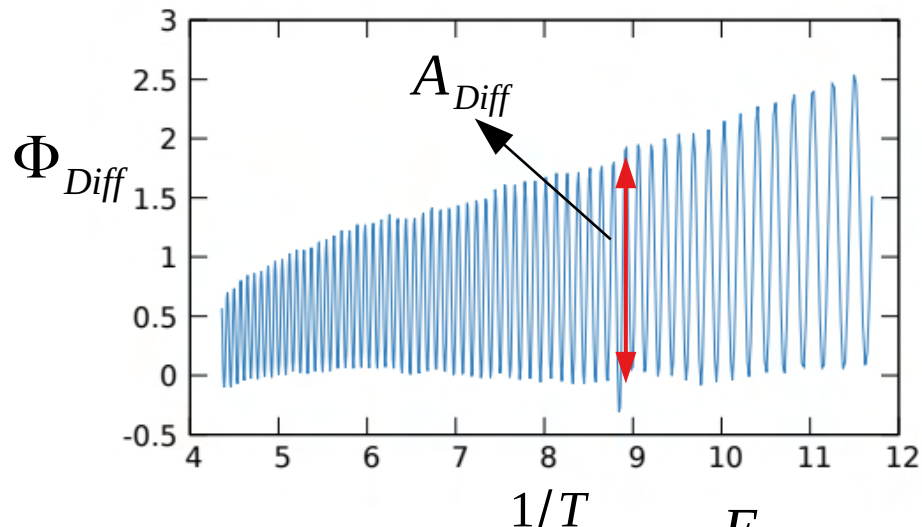
Thermomigration



➡ Slope Q

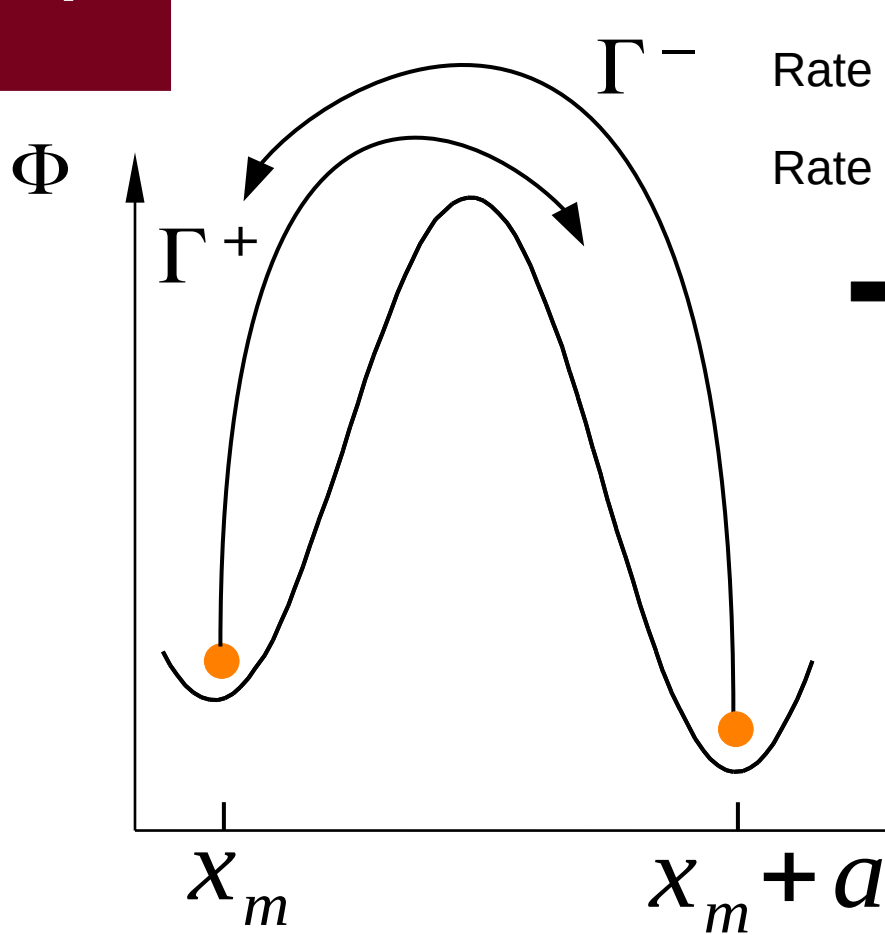
$$\Phi_{Diff} = \Phi + \frac{Q}{T}$$

Diffusion



➡ Amplitude $A_{Diff} \propto \frac{E_m}{T}$

➡ Study the kinetics of the adatom with parameters Q and A_{Diff}



Rate for jump to the right $\Gamma^+ = \nu^+ e^{-[\Phi(x_m + \frac{a}{2}) - \Phi(x_m)]}$

Rate for jump to the left $\Gamma^- = \nu^- e^{-[\Phi(x_m + \frac{a}{2}) - \Phi(x_m + a)]}$

➡ Average velocity of the adatom :

$$\langle v \rangle = a(\Gamma^+ - \Gamma^-)$$

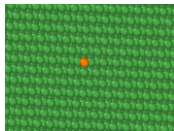
After a Taylor expansion to the first order:



$$\langle v \rangle = -\frac{D(T)}{k_B} e^{-a \frac{(Q - E_m)}{2 k_B T^2} \frac{\partial T}{\partial x}} \frac{Q}{k_B T^2} \frac{\partial T}{\partial x}$$

Diffusion coefficient : $D(T)$

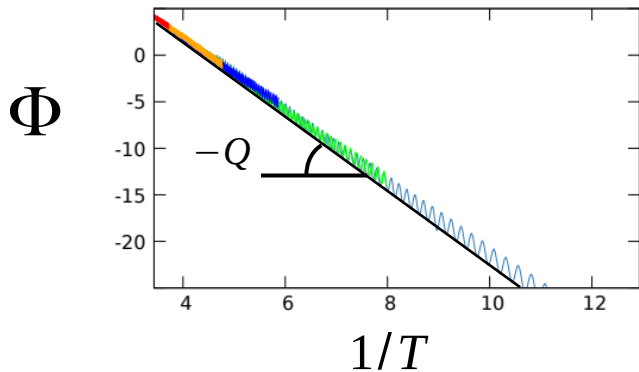
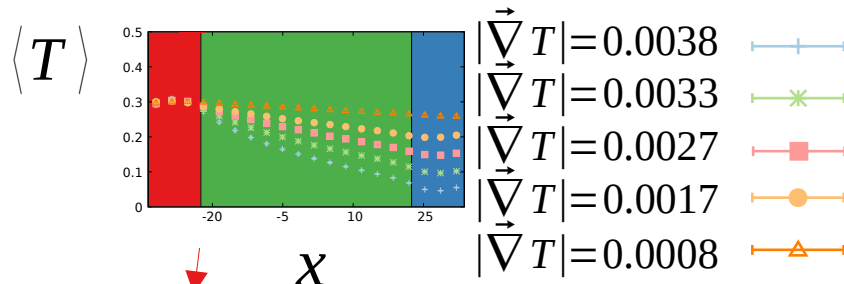
Mean square displacement on isothermal surfaces



$$\frac{dx}{dt} = - \frac{D(T)}{k_B} e^{-a \frac{(Q-E_m)}{2 k_B T^2} \frac{\partial T}{\partial x}}$$

The equation shows the relationship between the mean square displacement $\frac{dx}{dt}$ and the diffusion coefficient $D(T)$. The activation energy Q is circled in blue, and the temperature gradient term $\frac{\partial T}{\partial x}$ is circled in red. A blue arrow points from the Q term to the plot below, and a red arrow points from the $\frac{\partial T}{\partial x}$ term to the temperature profile plot above.

Temperature profil



Thermodynamic potential

KINETIC OF THERMOMIGRATION

Application of our new model on the trajectories of thermomigrations by integrating :

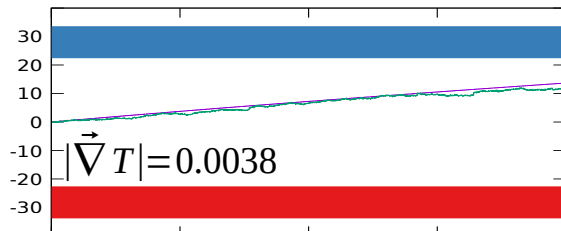
$$\frac{dx}{dt} = -\frac{D(T)}{k_B} e^{-a \frac{(Q-E_m)}{2k_B T^2} \frac{\partial T}{\partial x}} \frac{Q}{k_B T^2} \frac{\partial T}{\partial x}$$

➡ Model in agreement with average molecular dynamics trajectories

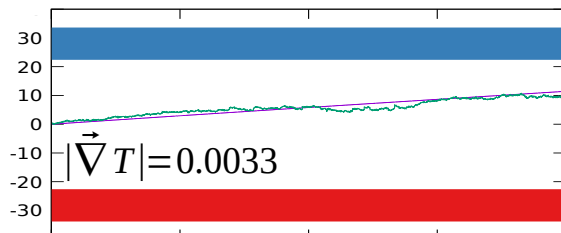
➡ This agreement validates our approach and hypotheses

— model
— average

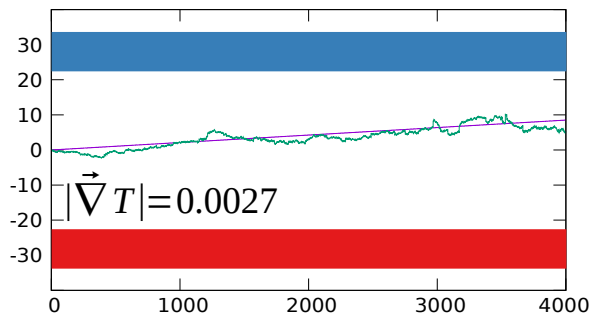
X



X



X



t

Application of thermodynamic integration to a cluster subjected to a thermal gradient

Extension of thermodynamic integration scheme to clusters

$$\frac{\partial(\Phi(x_0))}{\partial x} = - \left\langle \frac{Fx_{\text{adatom} \leftrightarrow \text{substrate}}}{T(x_0)} \right\rangle - \left\langle \frac{\frac{Ep_{\text{adatom} \leftrightarrow \text{substrate}}(x_0, y, z)}{2} + Ec_{\text{adatom}}(vx, vy, vz)}{T(x_0)^2} \frac{\partial T(x_0)}{\partial x} \right\rangle$$

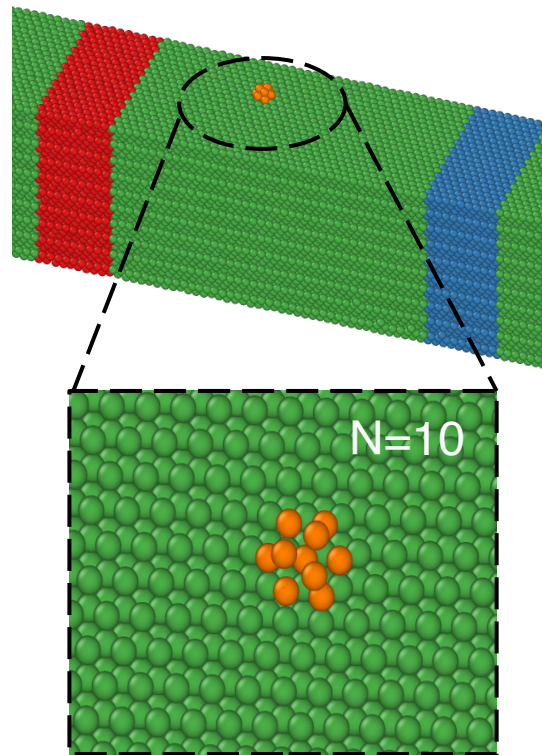
x_0 Position of the adatom



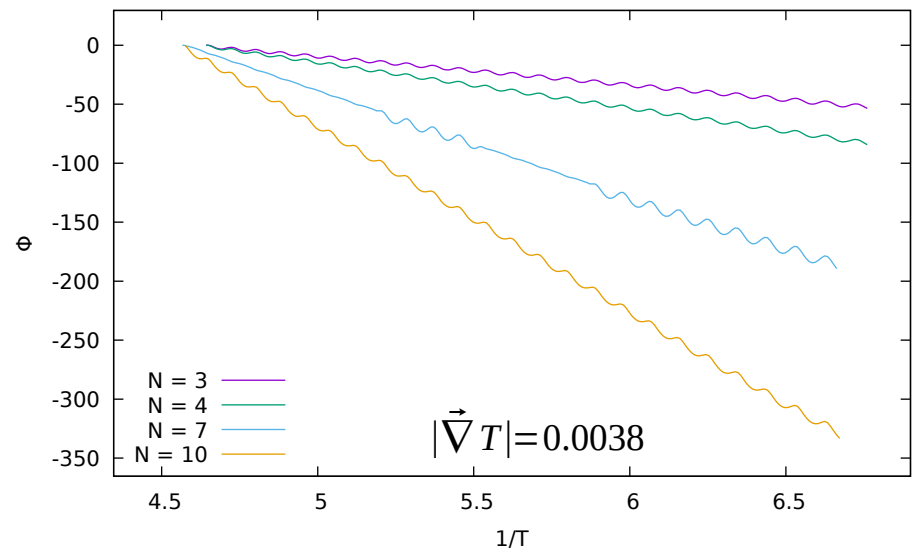
x_0 Center of mass of the cluster

$$\frac{\partial(\Phi(x_0))}{\partial x} = - \left\langle \frac{Fx_{\text{cluster} \leftrightarrow \text{substrate}}}{T(x_0)} \right\rangle - \left\langle \frac{Ep_{\text{cluster} \leftrightarrow \text{cluster}}(x_0, y, z) + \frac{Ep_{\text{cluster} \leftrightarrow \text{substrate}}(x_0, y, z)}{2} + Ec_{\text{cluster}}(vx, vy, vz)}{T(x_0)^2} \frac{\partial T(x_0)}{\partial x} \right\rangle$$

$$+ \left\langle \frac{1}{T(x_0)^2} \frac{\partial T(x_0)}{\partial x} \sum_k^M \left[x_k, \frac{Fx_{k \leftrightarrow \text{substrate}}}{2} + \sum_i^N x_i, \frac{Fx_{k \leftrightarrow i}}{2} \right] \right\rangle + \left\langle \frac{\partial^2 \frac{1}{T}(x_0)}{\partial x^2} \sum_k^M x_k, \left[\frac{p_k^2}{2m_k} + Ep_{k \leftrightarrow \text{substrate}} \right] \right\rangle$$



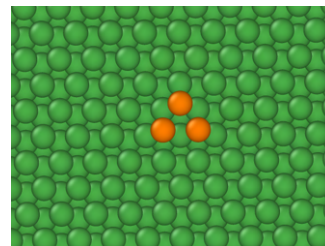
Thermodynamic integrations on cluster



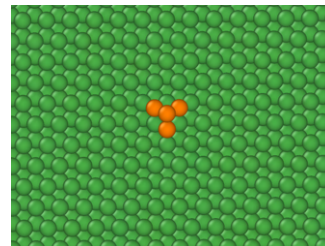
➡ effect of the size of the cluster

➡ effect of the conformation

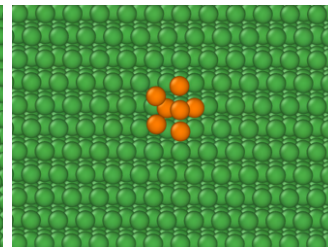
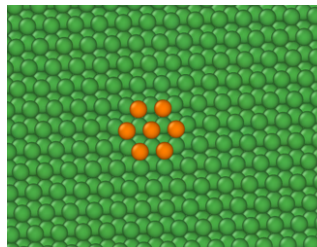
N=3



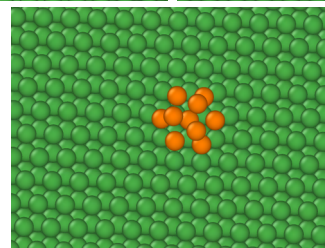
N=4



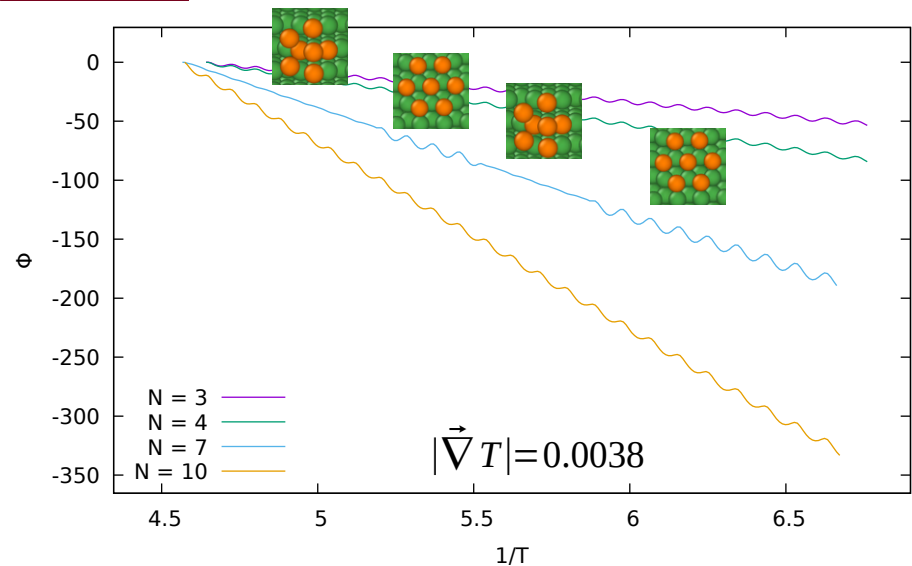
N=7



N=10



Thermodynamic integrations on cluster

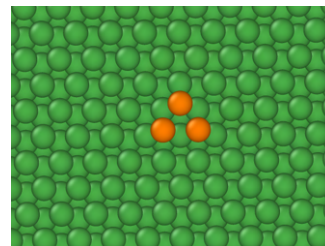


➡ effect of the size of the cluster

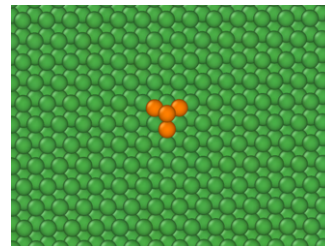
➡ effect of the conformation

Work in progress ...

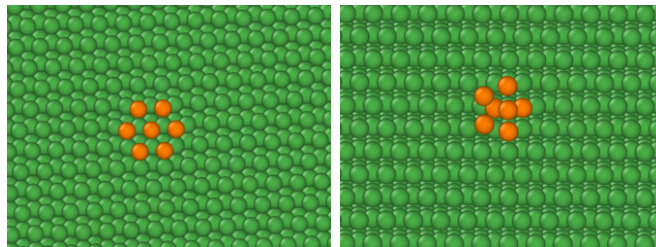
$N=3$



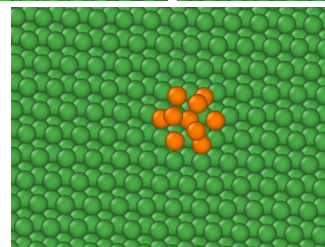
$N=4$



$N=7$



$N=10$



Conclusion

➡ Simulations and characterizations of thermomigration

➡ Generalization of thermodynamic integration to measure the thermodynamic potential

Drift

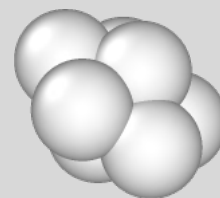
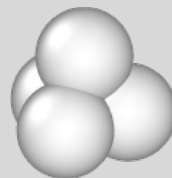
Diffusion

➡ Transition state theory in the case of a system subjected to a thermal gradient to deduce a drift velocity

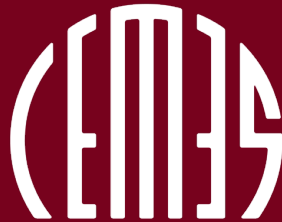
➡ Preliminary results on clusters

Perspectives:

➡ Elastic effects on clusters thermomigration

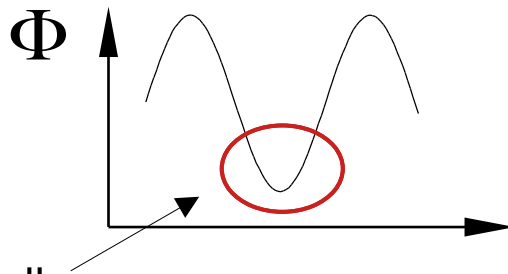


Thank you for your attention !



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$V_0 > 0$ Binding energy

➔ At the minima of a crystalline potential well

➔ Harmonic approximation

➔ potential energy of the adatom

$$\langle Ep(x_0) \rangle = -V_0 + 2 \frac{T(x_0)}{2}$$

➔ kinetic energy of the adatom

$$\langle Ek(x_0) \rangle = \frac{3}{2} T(x_0)$$

$$\frac{\partial(\Phi(x_0))}{\partial x} = \left\langle \frac{\frac{\partial Ep(x_0, y, z)}{\partial x}}{T(x_0)} \right\rangle - \left\langle \frac{\frac{1}{2} Ep(x_0, y, z) + Ec(v_x, v_y, v_z)}{T(x_0)^2} \frac{\partial T(x_0)}{\partial x} \right\rangle$$

$$\Phi(x_0)_{min} = \frac{-V_0}{2T(x_0)} - 2 \ln(T(x_0)) = \frac{-Q}{T(x_0)} - 2 \ln(T(x_0))$$

2

ORDER OF MAGNITUDE

Lennard-Jones potential :

$$E_{LJ}(r_{ij}) = 4 \epsilon \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^6 \right] \longrightarrow$$

Lennard-Jones reduced units :

$$r^+ = \frac{r}{\sigma} \quad t^+ = t \left(\frac{\epsilon}{m \sigma^2} \right)^{\frac{1}{2}}$$

$$T^+ = \frac{T k_B}{\epsilon} \quad E^+ = \frac{E}{\epsilon}$$

Using copper's parameter :

$$\sigma_{Cu} = 0.2338 \text{ nm}$$

$$\epsilon_{Cu} = 0.4057 \text{ eV}$$

$$m_{Cu} = 105.4910 \cdot 10^{-27} \text{ kg}$$



$$|\vec{\nabla} T| = 0.0038 = 76.5 \text{ K/nm} \quad Q = 1.9 = 0.77 \text{ eV}$$

$$|\vec{\nabla} T| = 0.0027 = 54.37 \text{ K/nm} \quad E_m = 0.23 = 0.09 \text{ eV}$$

$$T_{HOT} = 0.3 = 1412.4 \text{ K}$$

$$t = 25000 = 7.45 \text{ ns}$$

$$T_{COLD} = 0.05 = 235.4 \text{ K}$$

$$Lx = 44.9 = 10.5 \text{ nm}$$

DIFFUSION

➡ Trajectory on the surface is diffusive

➡ No Lévy flights

