

Nanoscale Heat Radiation II

Conduction-radiation coupling and many-body effects

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Fréjus, 11 May 2022

Outline

- Conduction-radiation coupling
- Many-body effects

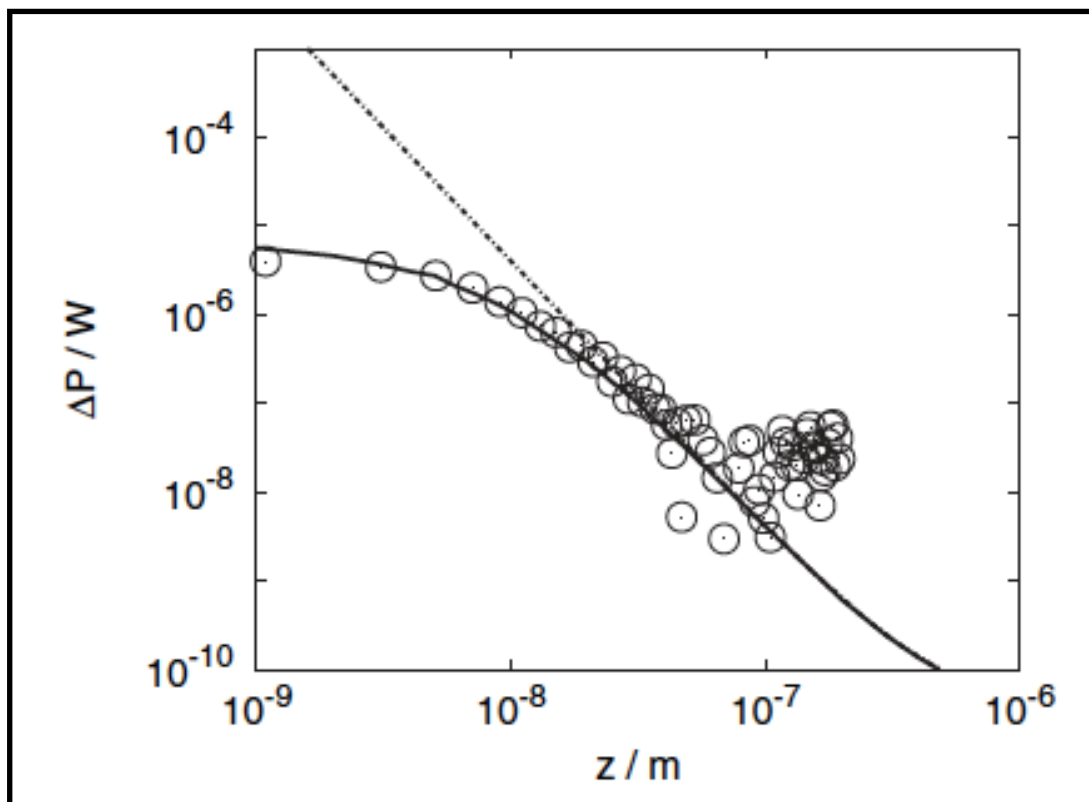
Outline

- **Conduction-radiation coupling**
- Many-body effects

Flux divergence at small distance

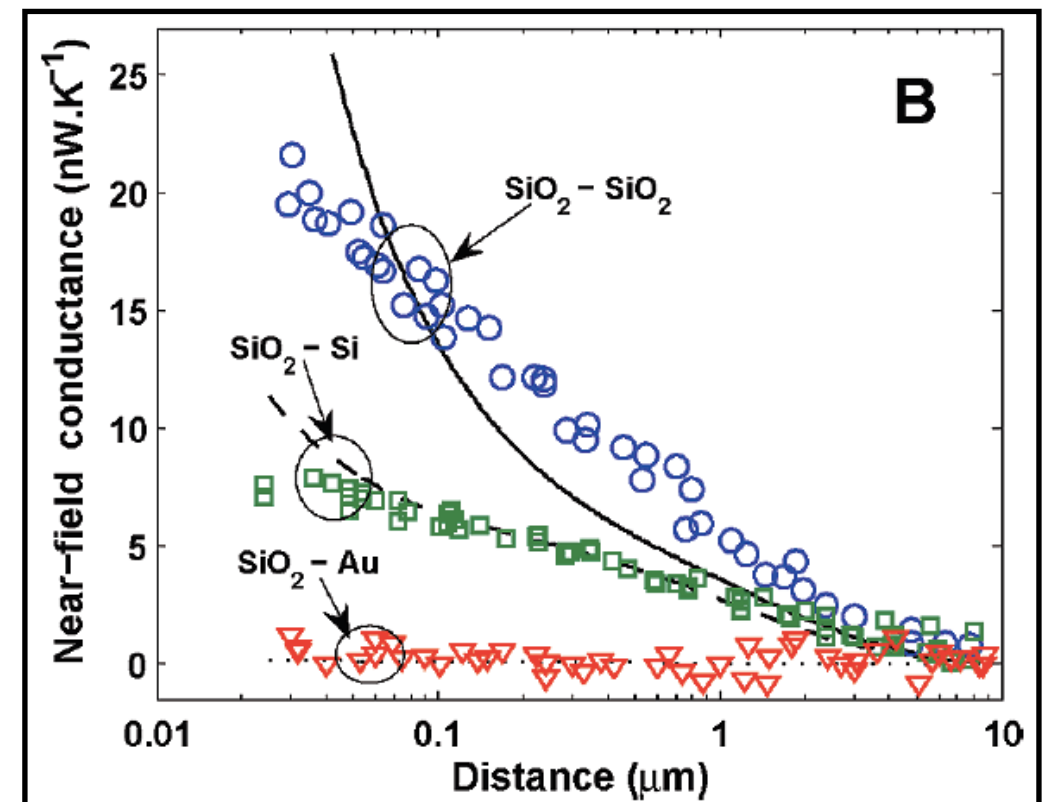
Some **experiments** have questioned the flux **divergence**

$$d \lesssim 1 \text{ nm}$$



A. Kittel et al., PRL **95**, 224301 (2005)
K. Kloppstech et al., Nat. Commun. **8**, 14475 (2017)

$$d \simeq 70 \text{ nm}$$



S. Shen et al., Nano Letters **9**, 2909 (2009)

Some theoretical suggestions

Role of non-local effects

- *Ad hoc* modification of fluctuation-dissipation theorem with a cutoff
- The effect plays a role for $d < 10$ nm

K. Joulain, JQRST **109**, 294 (2008)

P.-O. Chapuis et al, PRB **77**, 035431 (2008)

Coupling with conduction?

- Studied only in the specific case of **silicon**
- Coupling induces a tiny temperature gradient

B.T. Wong et al., JQRST **143**, 46 (2014)

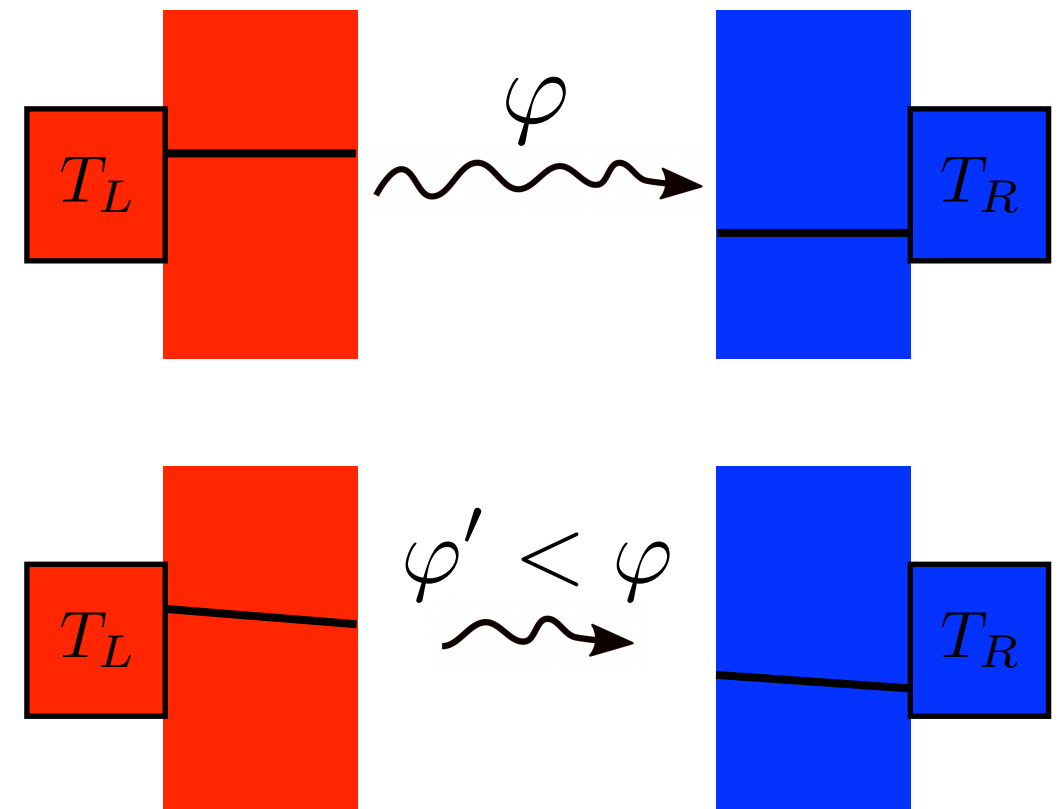
No temperature profile

Conventional Polder - van Hove theory

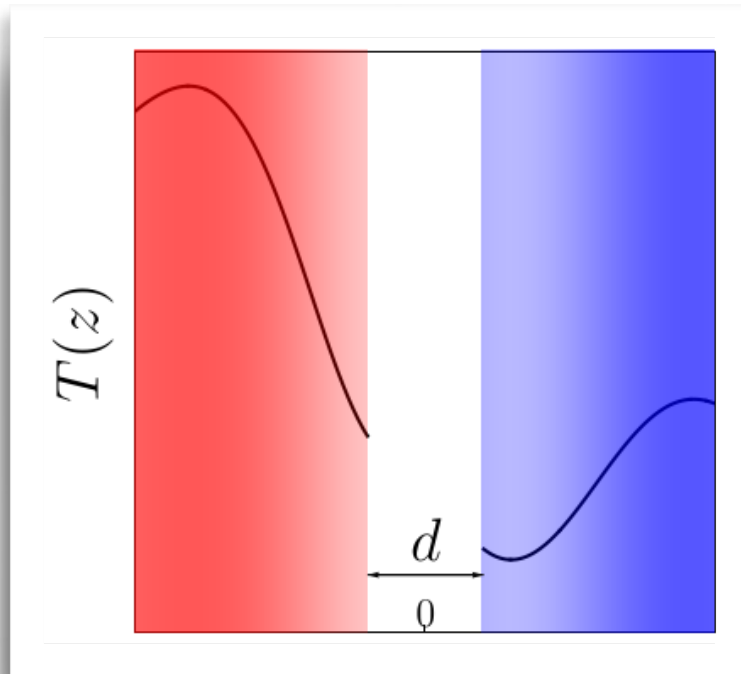
VS

Temperature profile

Modified (reduced) radiative heat flux



The role of a temperature profile

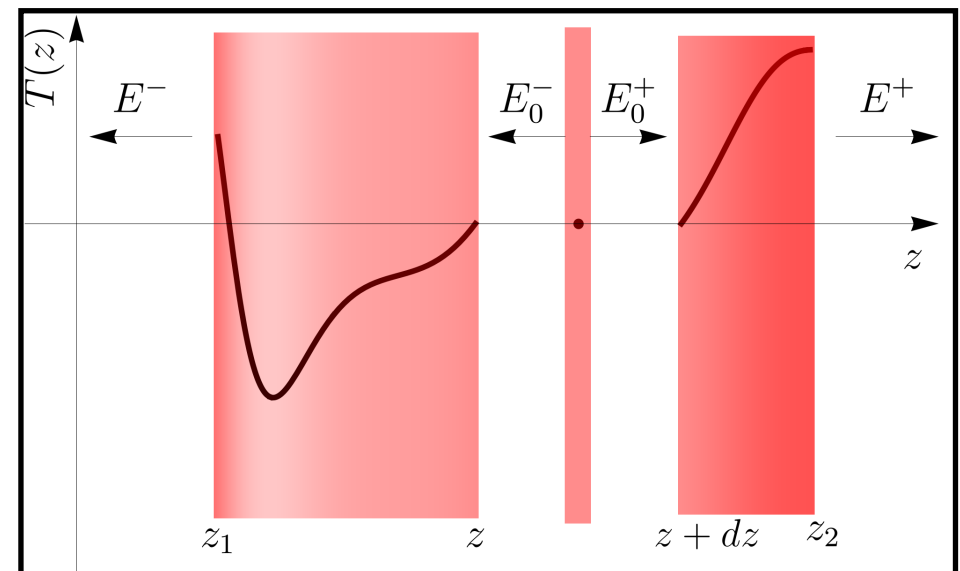


Two planar parallel slabs

- Translational invariance
- Dielectric properties
- Distance
- **Arbitrary temperature profile**

One slab having $T(z)$

- **Generalized N - body problem**
- Contribution of each layer of thickness dz
- Statistical description of E^+ and E^-



Imposed temperature profiles can lead to modified flux asymptotic laws

$$\varphi \simeq d^{-2} \quad \varphi \simeq d^{-1} \quad \varphi \simeq \log(\alpha d) \quad \varphi \simeq \text{const.}$$

Coupling in the Fourier regime

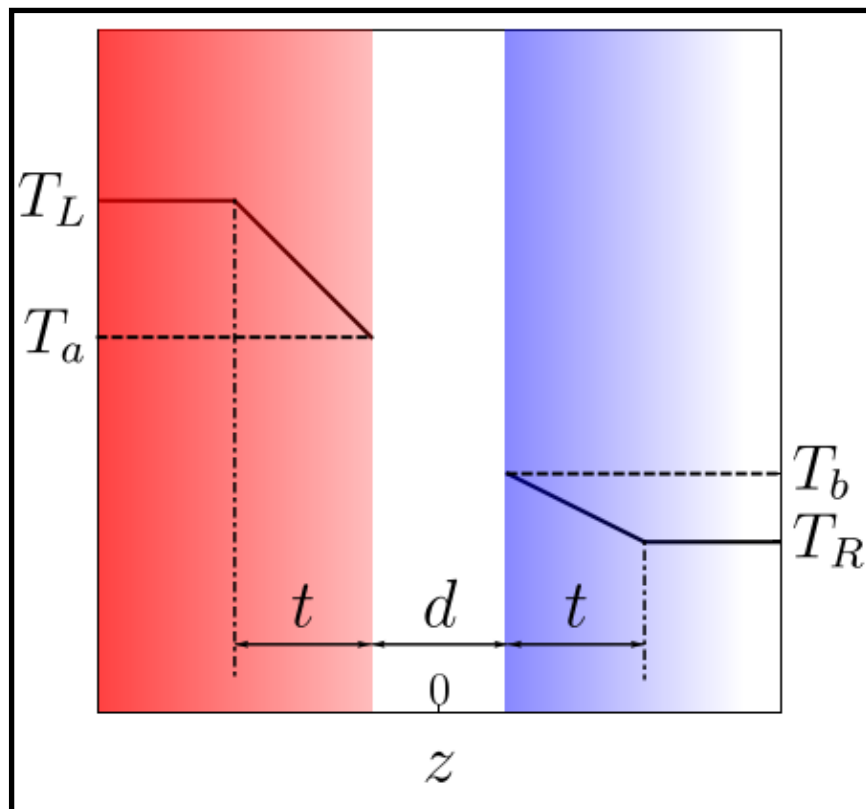
Can **conduction-radiation coupling** induce an observable temperature profile ?

Coupled conduction-radiation equation

$$\frac{\partial}{\partial z} \left[\kappa(z) \frac{\partial}{\partial z} T(z) \right] + \int dz' \varphi(z', z) = Q(z)$$

$\kappa(z)$ thermal conductivity $T(z)$ temperature profile

$\varphi(z', z)$ radiative flux $z' \rightarrow z$ $Q(z)$ external source



Assumptions

Fourier regime

The temperature profile is linear in each region

Thermostats at T_L and T_R

T_a and T_b : interface temperatures

Simplified radiative flux $\varphi \simeq h_0 \frac{T_a - T_b}{d^2}$

Closed-form solutions

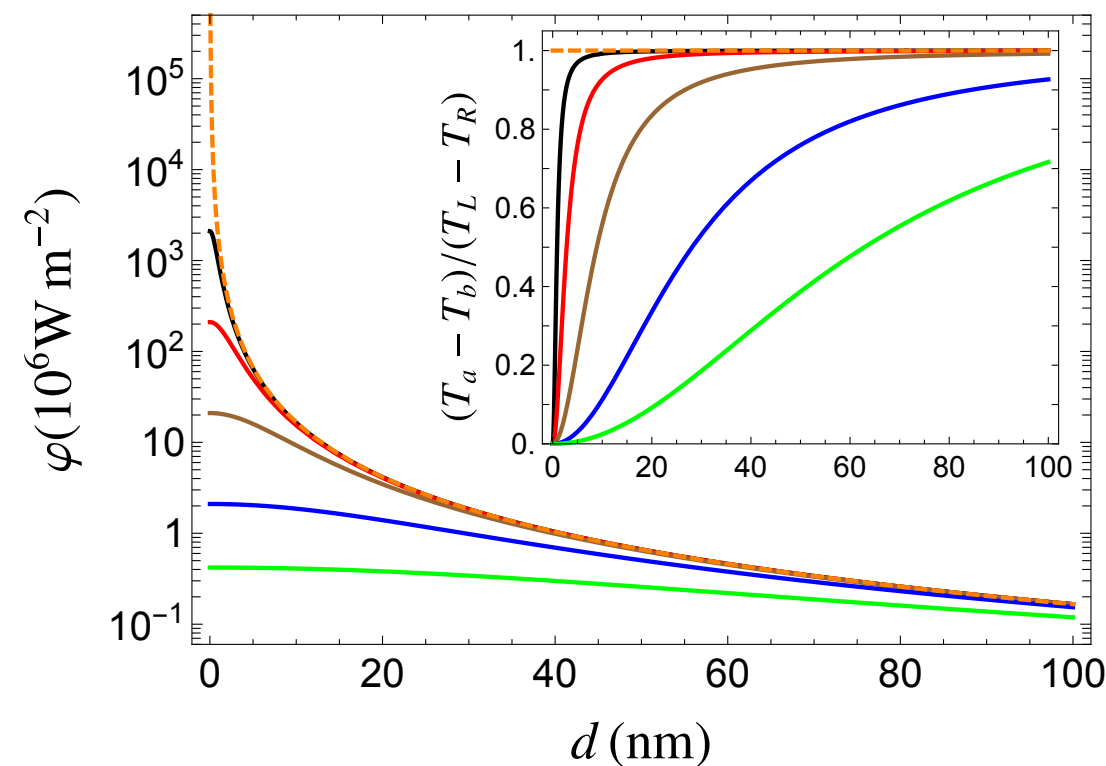
$$T_a - T_b = \frac{\kappa d^2 (T_L - T_R)}{\kappa d^2 + 2th_0}$$

$$\varphi = \frac{\kappa h_0 (T_L - T_R)}{\kappa d^2 + 2th_0}$$

R. Messina et al., Phys. Rev. B **94**, 121410(R) (2016)

Impact of material properties

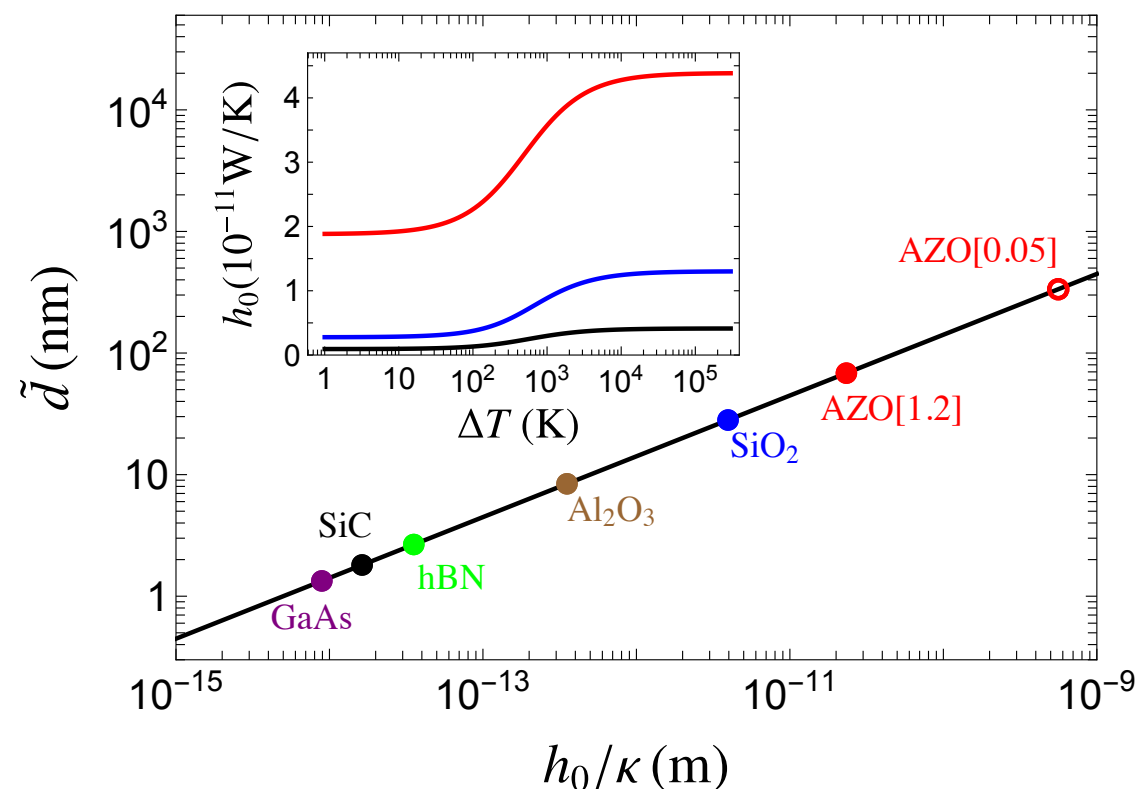
- Two SiO₂ slabs
(T_L, T_R) = (600, 300) K
- Thicknesses $t = 100$ nm, $1\ \mu\text{m}$, $10\ \mu\text{m}$,
 $100\ \mu\text{m}$, $500\ \mu\text{m}$
- Deviations at **tens of nanometers**
- **Flux saturation**



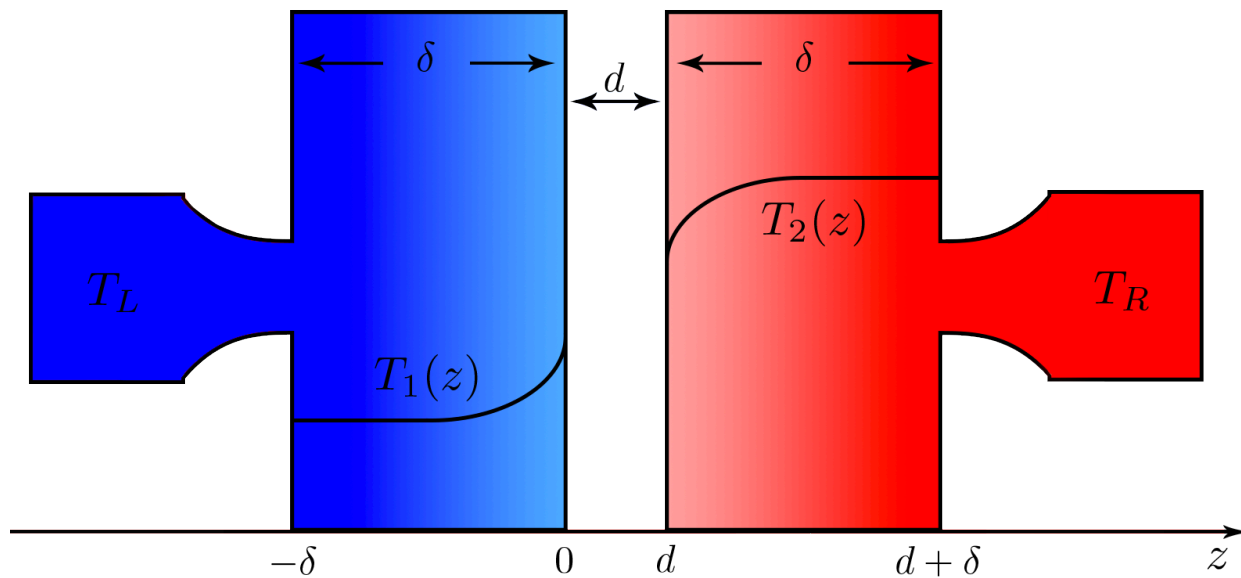
Critical distance

$$\tilde{d} = \sqrt{2th_0/\kappa}$$

$$T_a - T_b = (T_L - T_R)/2$$



Coupling beyond Fourier regime



Boltzmann transport equation

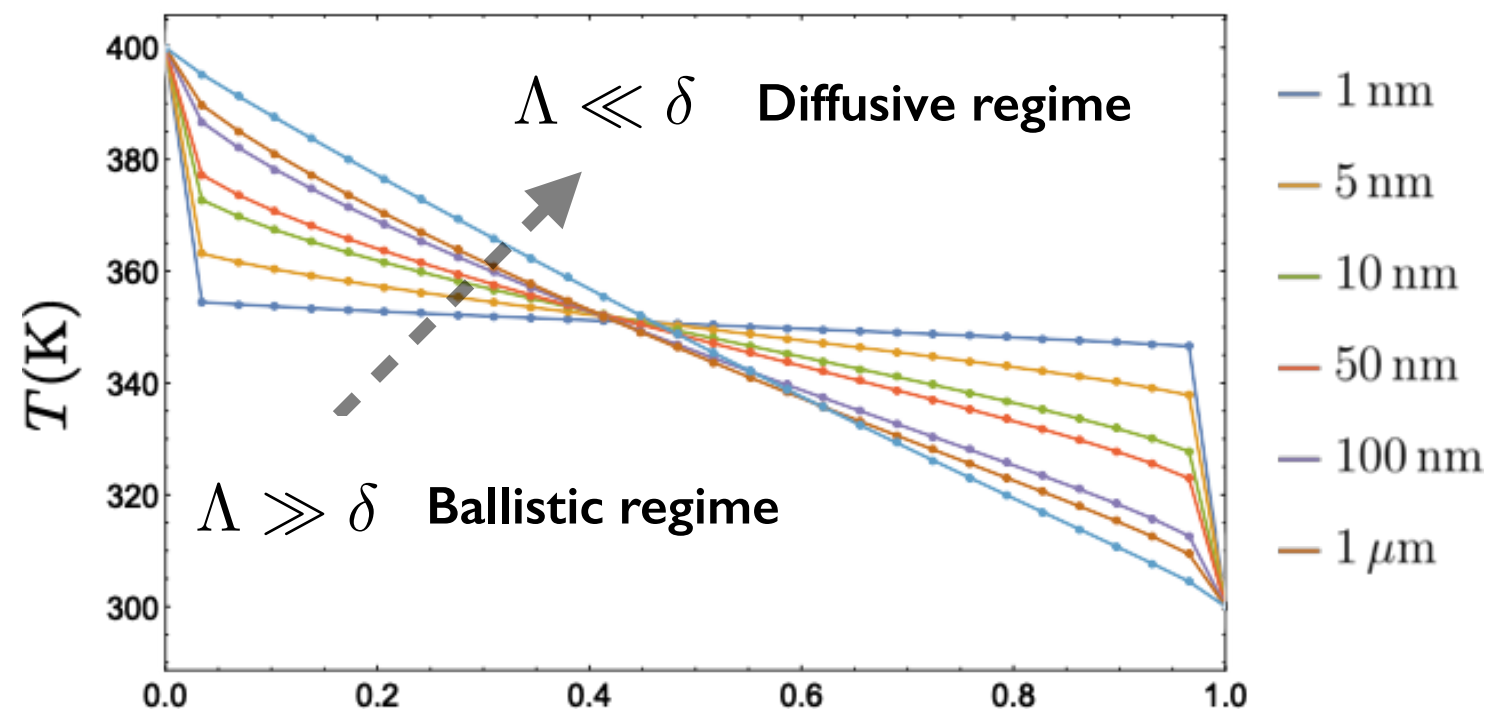
$$\frac{\partial f_p}{\partial t} + \mathbf{v}_{g,p} \cdot \nabla f_p = -\frac{f_p - f_0}{\tau_p(\omega, T(\mathbf{r}))}$$

+

Energy conservation equation

$$\frac{\partial u(\mathbf{r}, t)}{\partial t} = P_{\text{rad}}(\mathbf{r}, t) + \nabla \cdot \varphi_{\text{cond}}(\mathbf{r}, t)$$

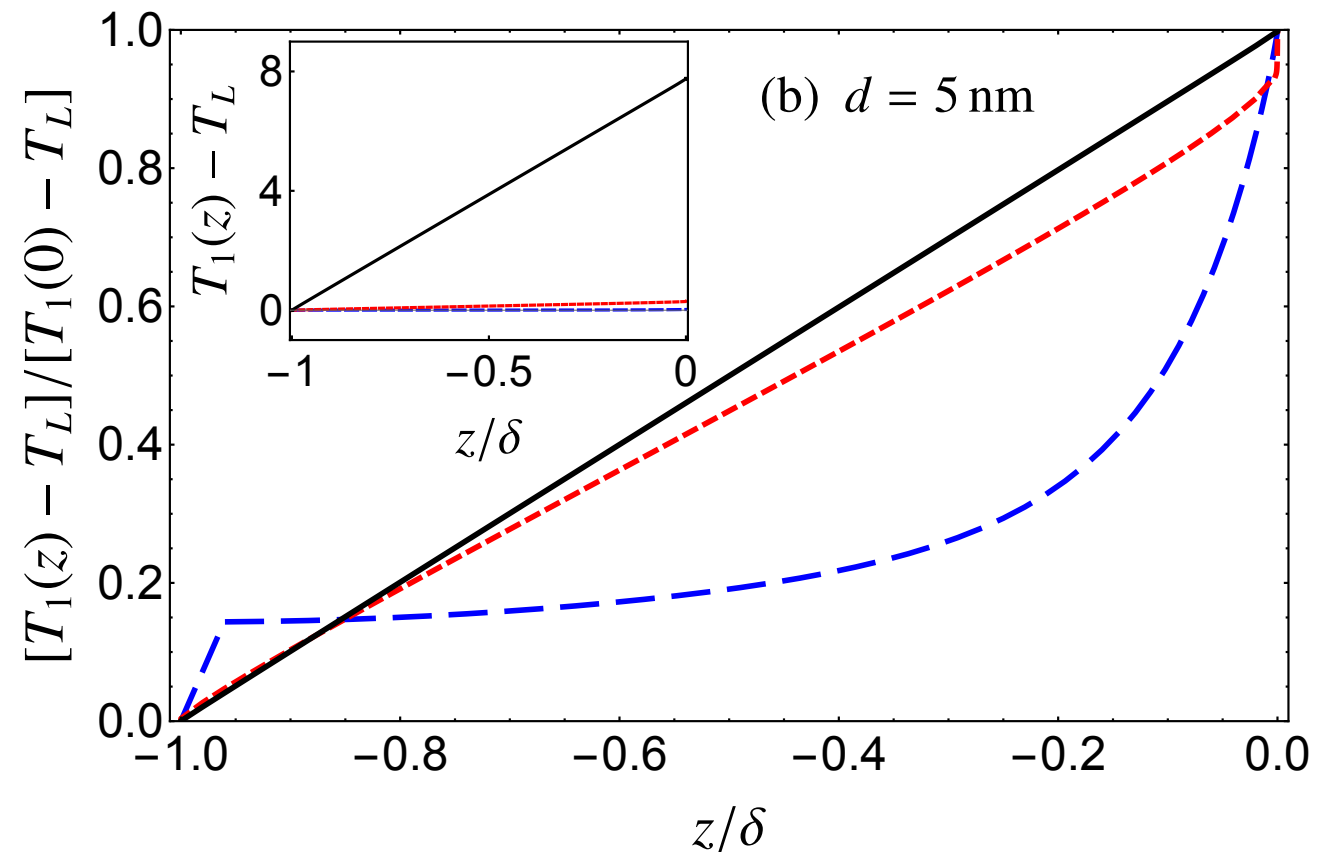
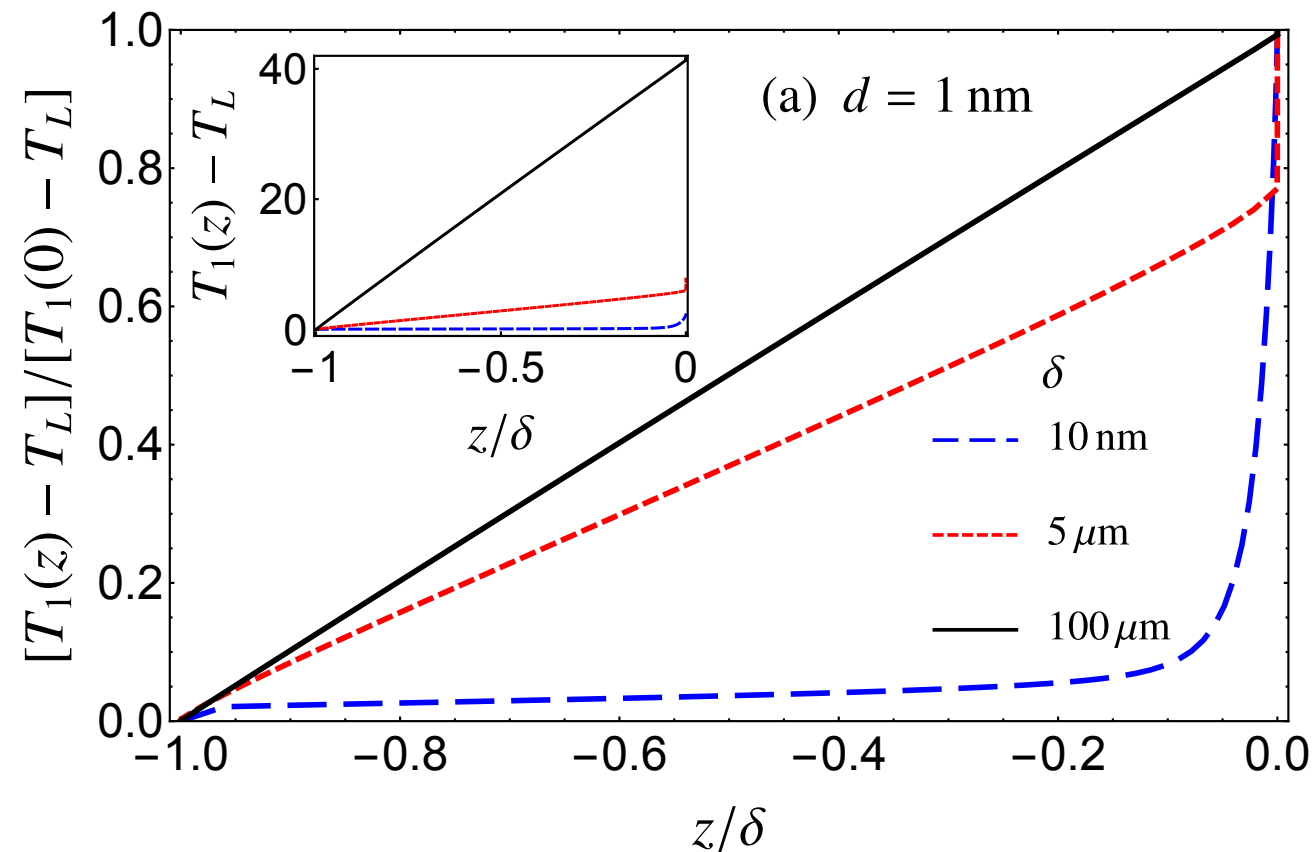
$$\varphi_{\text{cond}}(t, \mathbf{r}) = \sum_p \int_{4\pi} d\Omega \int d\omega \hbar\omega \mathbf{v}_{g,p}(\omega) f_p(t, \omega, \mathbf{r}, \Omega) \frac{D_p(\omega)}{4\pi}$$



Temperature profiles

Main figures: relative temperature gradient

Insets: absolute temperature gradient



Key points

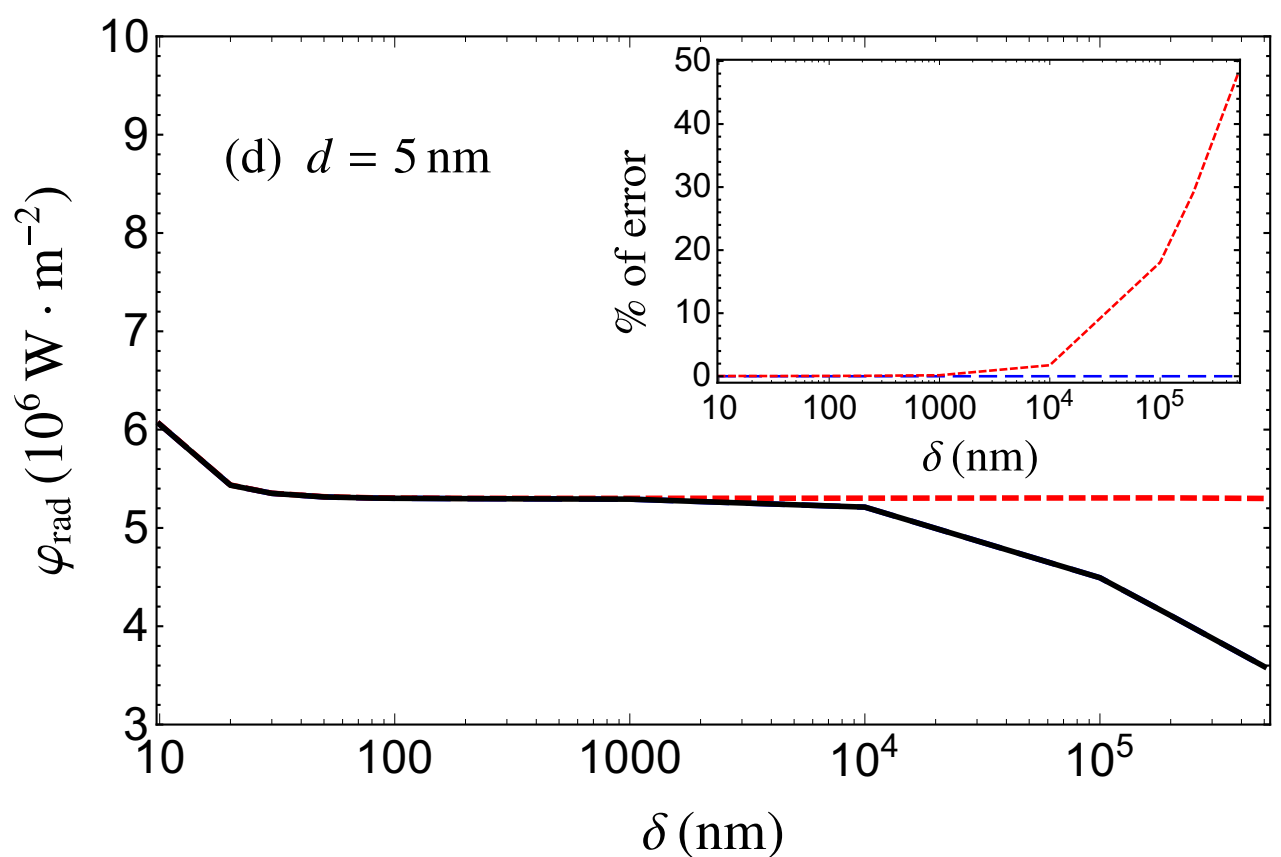
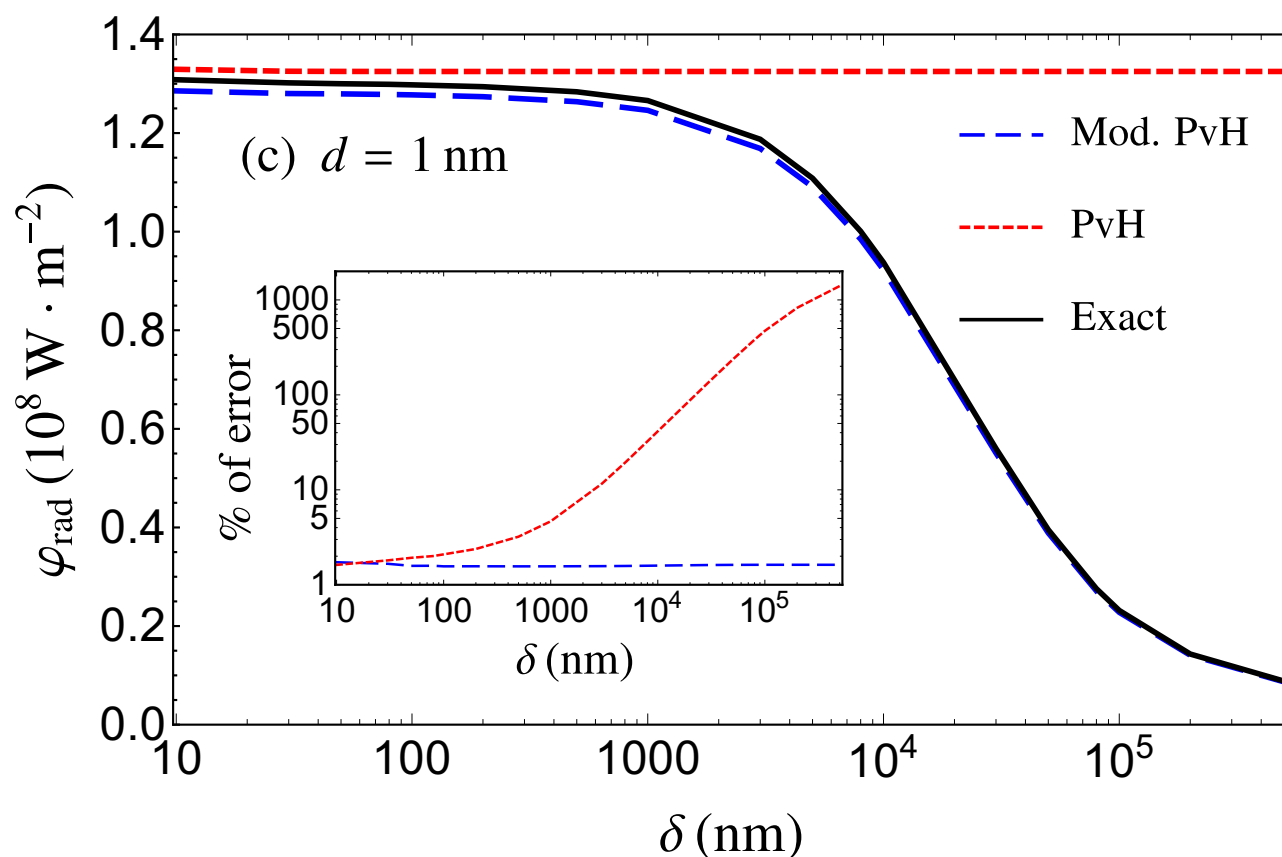
- The coupling produces a temperature profile within each body
- The temperature gradient and the profile shape strongly depend on d and δ

M. Reina, RM, and P. Ben-Abdallah, Phys. Rev. Lett. **125**, 224302 (2020)

Impact on radiative flux

Main figures: radiative heat flux

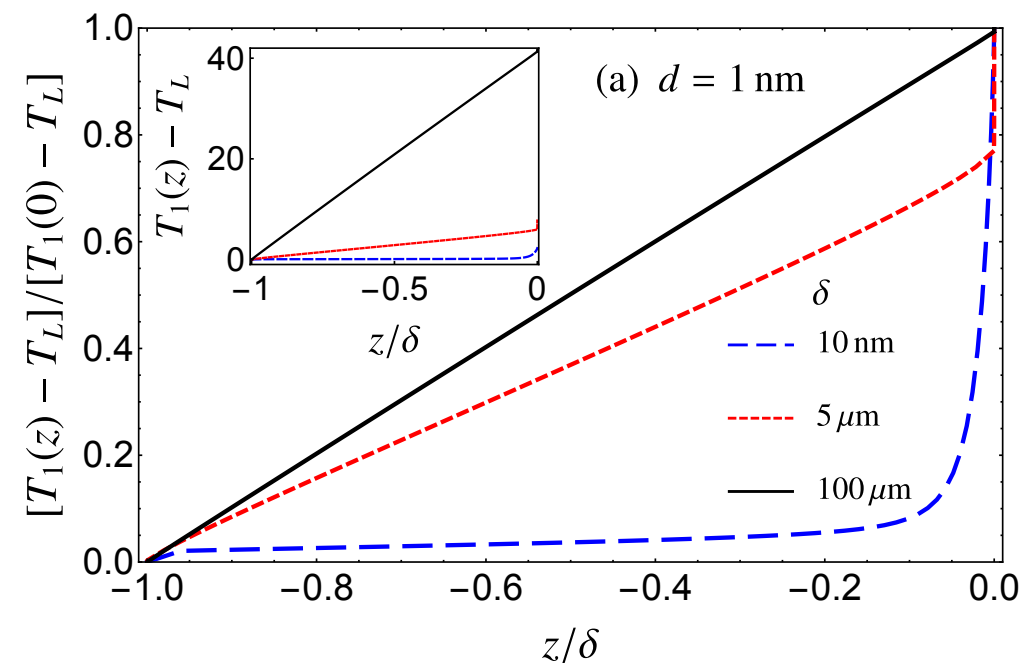
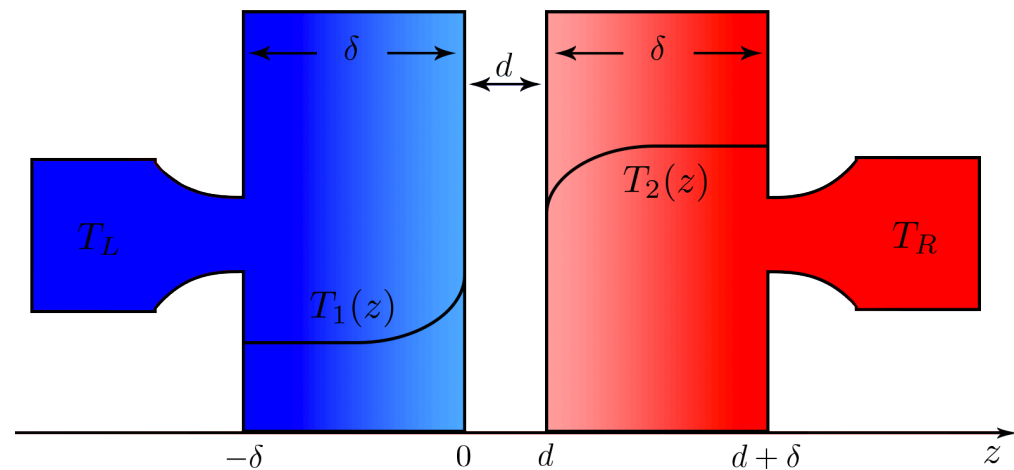
Insets: relative error



Key points

- In the case of **large thickness** and **small distance**, the exact radiative flux exact is significantly different respect to the case in which the coupling is neglected

Conduction-radiation coupling



- **Observable temperature profile** within each solid
- The effect is remarkable for **large slab thickness** (Fourier regime) and **small distance**
- Strong **reduction of exchanged radiative heat flux** with respect to conventional theory

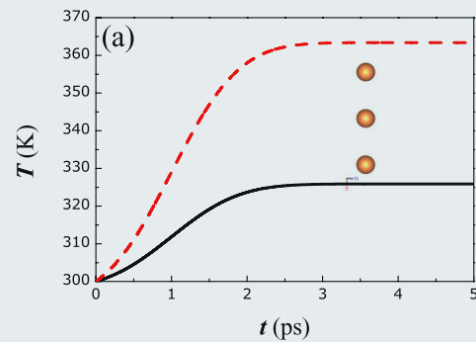
R. Messina et al., Phys. Rev. B **94**, 121410(R) (2016)

M. Reina et al., PRL **125**, 224302 (2020)

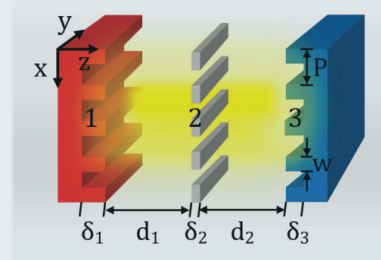
Outline

- Radiation-conduction coupling
- **Many-body effects**

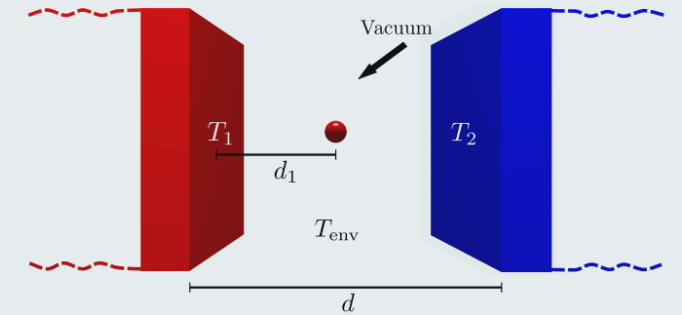
Recent interest in many-body effects



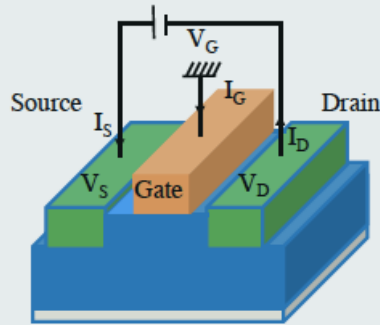
V. Yannopapas et al., PRL **110**, 044302 (2013)



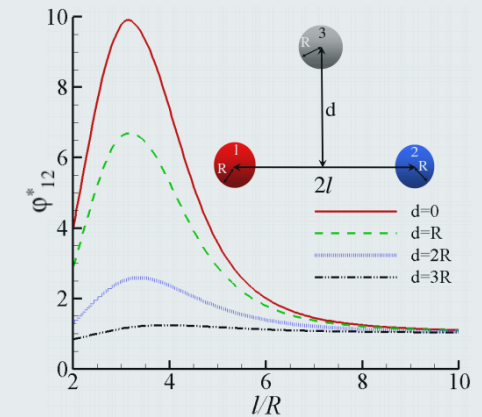
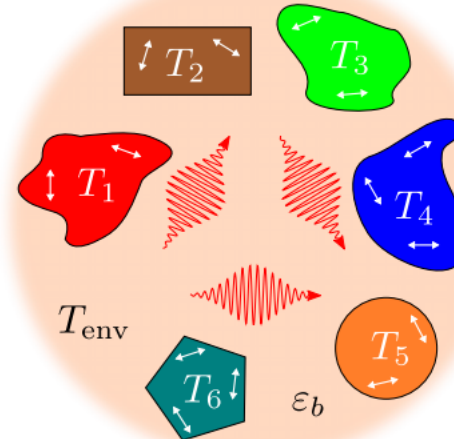
Y. H. Kan et al., PRB **99**, 035433 (2019)



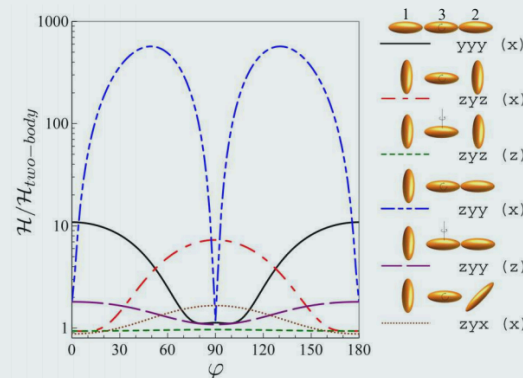
B. Müller et al., PRB **95**, 085413 (2017)



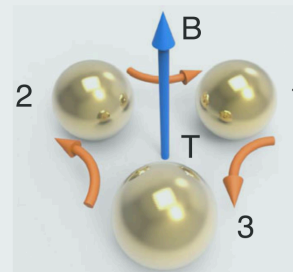
P. Ben-Abdallah and S.-A. Biehs, PRL **112**, 044301 (2014)



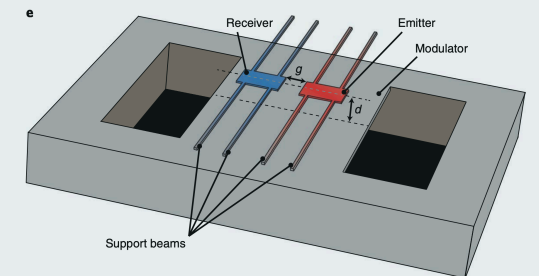
P. Ben-Abdallah et al., PRL **107**, 114301 (2011)



M. Nikbakht, J. Appl. Phys. **116**, 094307 (2014)



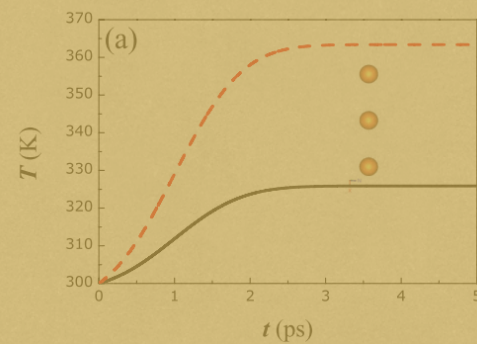
L. Zhu and S. Fan, PRL **117**, 134303 (2016)



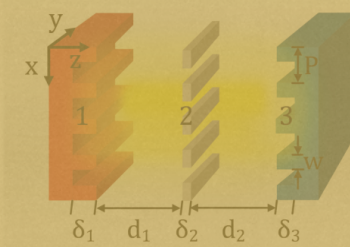
D. Thomson et al.,
Nat. Nanotechnol. **15**, 99 (2020)

S.-A. Biehs et al., Rev. Mod. Phys. **93**, 025009 (2021)

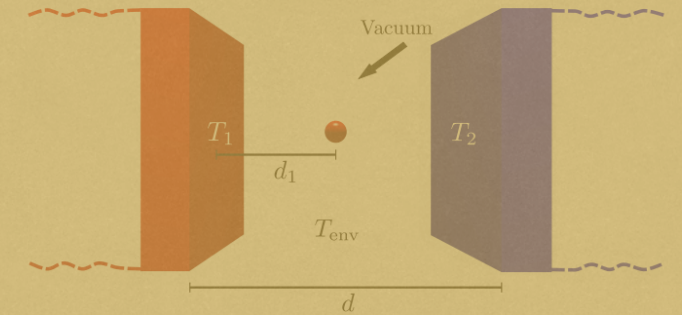
Recent interest in many-body effects



V. Yannopapas et al., PRL **110**, 084601 (2013)



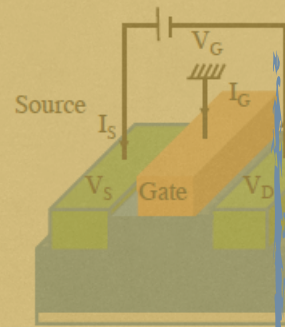
Y. H. Kan et al., PRB **99**, 035433 (2019)



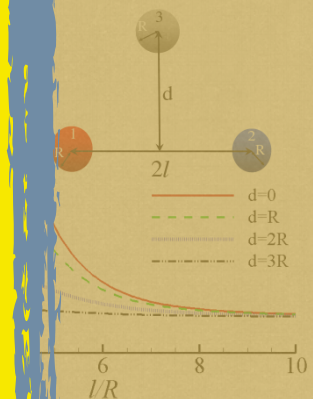
B. Müller et al., PRB **95**, 085413 (2017)

Main features

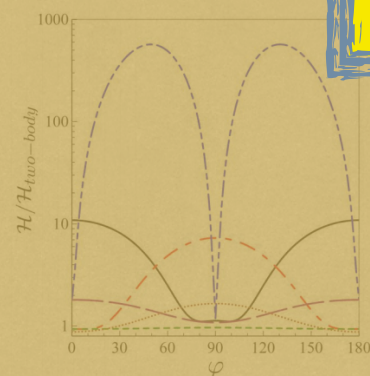
- Additional degrees of freedom
- Prediction of **purely many-body effects** (failure of additive model)
- Flux **amplification and modulation**



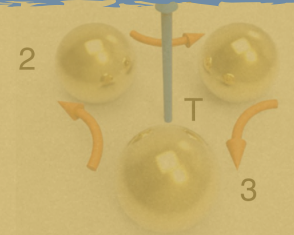
P. Ben-Abdallah et al., PRL **107**, 114301 (2011)



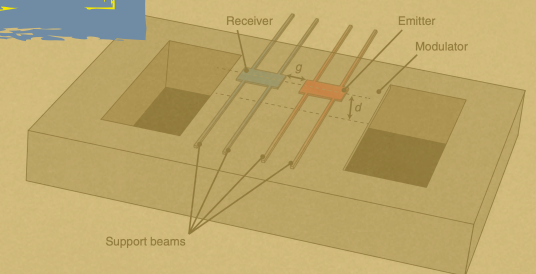
PRL **107**, 114301 (2011)



M. Nikbakht, J. Appl. Phys. **116**, 094307 (2014)



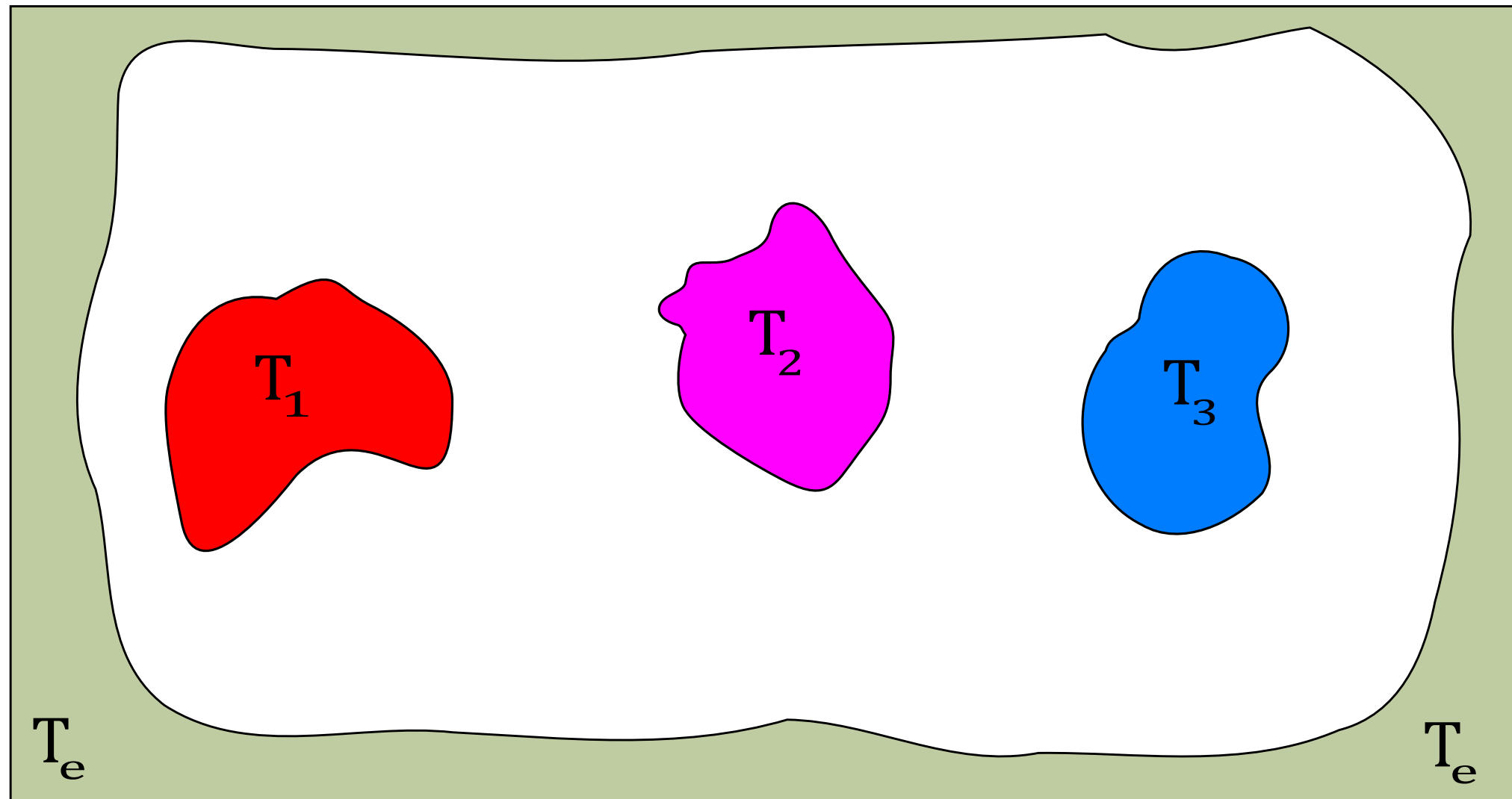
L. Zhu and S. Fan, PRL **117**, 134303 (2016)



D. Thomson et al., Nat. Nanotechnol. **15**, 99 (2020)

S.-A. Biehs et al., Rev. Mod. Phys. **93**, 025009 (2021)

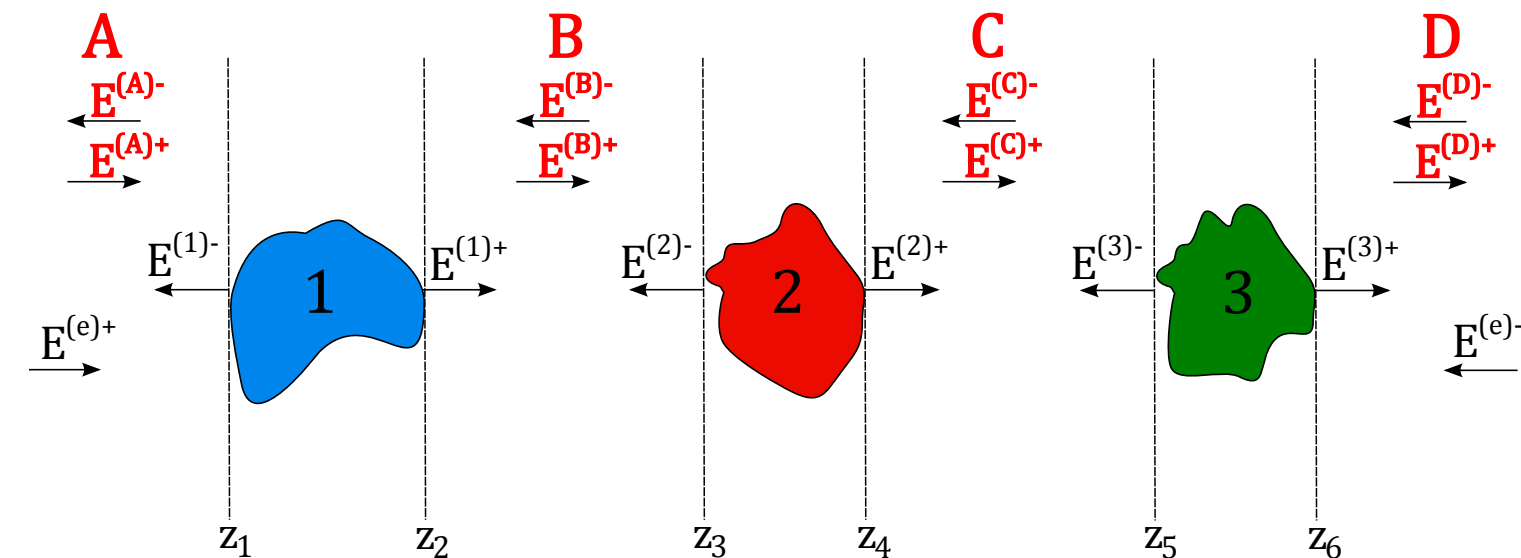
Three bodies: theoretical framework



- Three bodies having **arbitrary shape** and **material properties**
- Three **arbitrary temperatures** T_1 , T_2 and T_3
- **Environment** enclosing the three bodies at temperature T_e

Local thermal equilibrium

Sources of electromagnetic field



- Field $E^{(i)}$ emitted by **each body**
- **Environmental** field $E^{(e)}$

To be determined

- Properties of the source fields
- Passage from source fields to the total field in each region

The fluctuation-dissipation theorem is not applicable

The temperatures T_1, T_2, T_3 and T_e remain constant



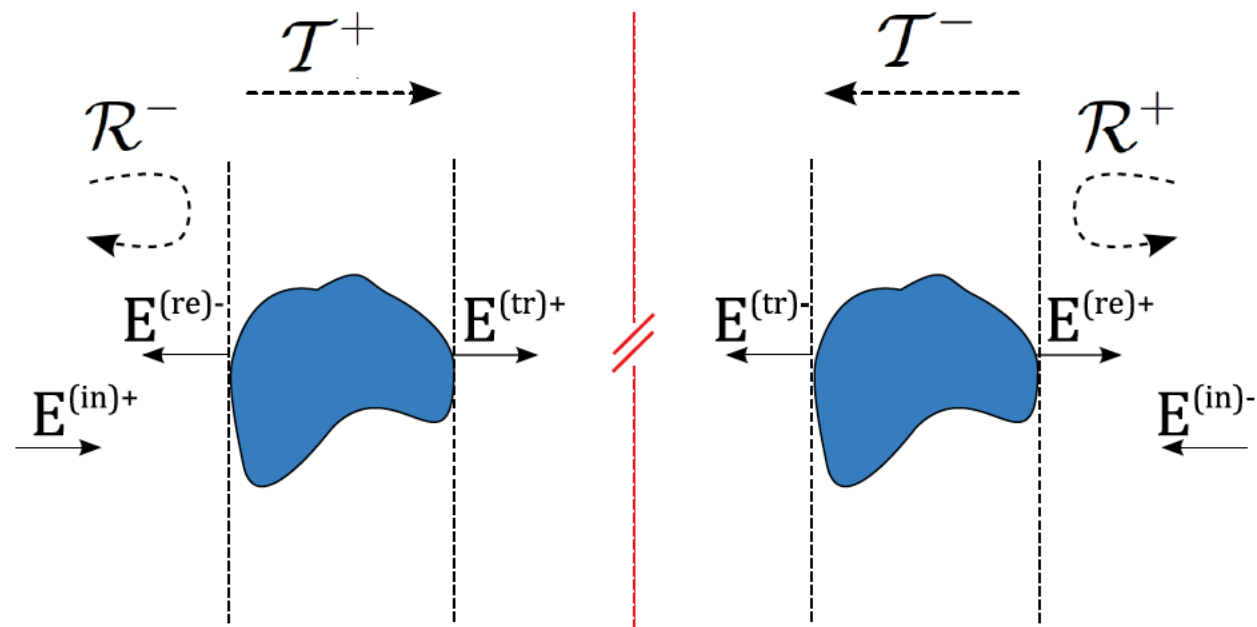
The correlation functions of the source fields are the same each individual body would have at **thermal equilibrium**

Main steps of the calculation

- Correlation functions of the source fields

$$\langle E_p^{(i)\phi}(\mathbf{k}, \omega) E_{p'}^{(i)\phi\dagger}(\mathbf{k}', \omega') \rangle$$

- Total field** in each region expressed as a function of source fields through **scattering operators**



2 bodies: RM and M. Antezza, EPL **95**, 61002 (2011), PRA **84**, 042102 (2011)

3 bodies: RM and M. Antezza, PRA **89**, 052104 (2014)

N planar slabs: I. Latella et al., PRB **95**, 205404 (2017)

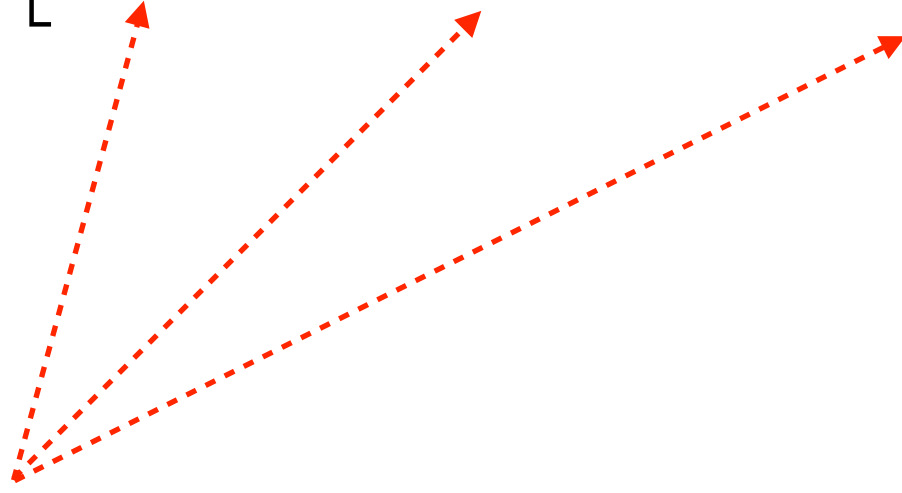
Heat transfer on body 1

Exchange with environment, body 2 and 3

$$H_1 = \text{Tr} \left\{ \hbar\omega \left[n_{e1} A_{e1} + n_{21} A_{21} + n_{31} A_{31} \right] \right\}$$

Bose-Einstein
populations

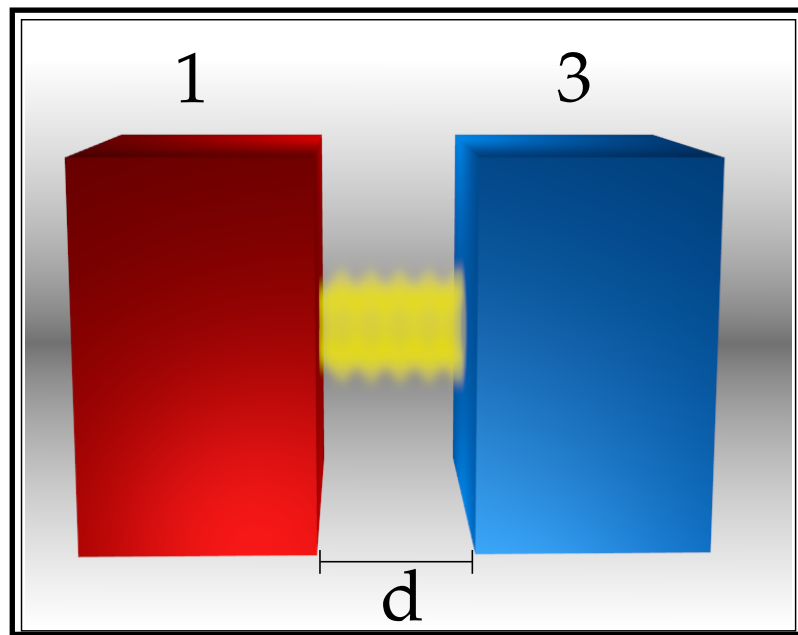
$$n_{ij} = n(\omega, T_i) - n(\omega, T_j)$$



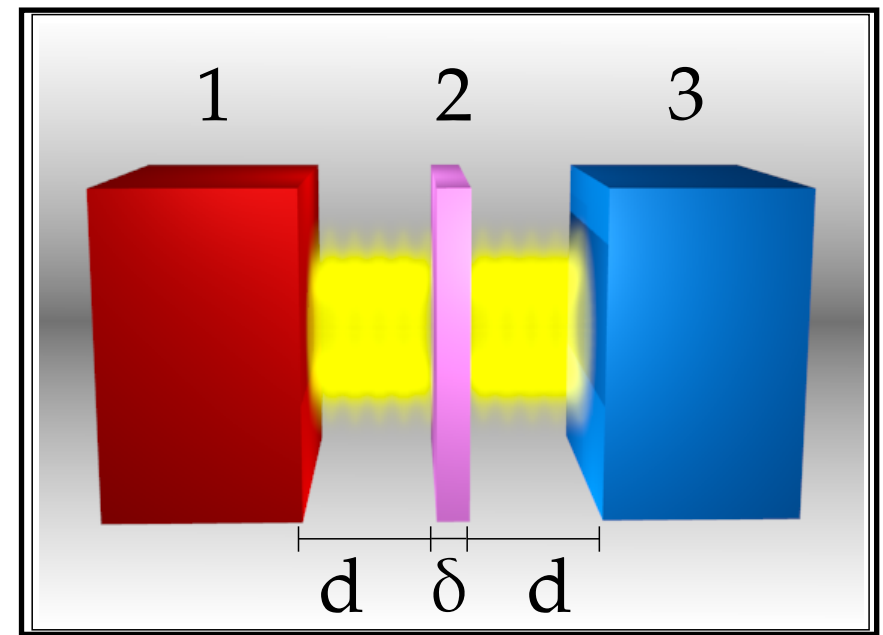
Many-body effect: each energy exchange depends on the presence of the other bodies!

Amplification of heat transfer

We want to compare



$$\varphi_{2s}(d)$$



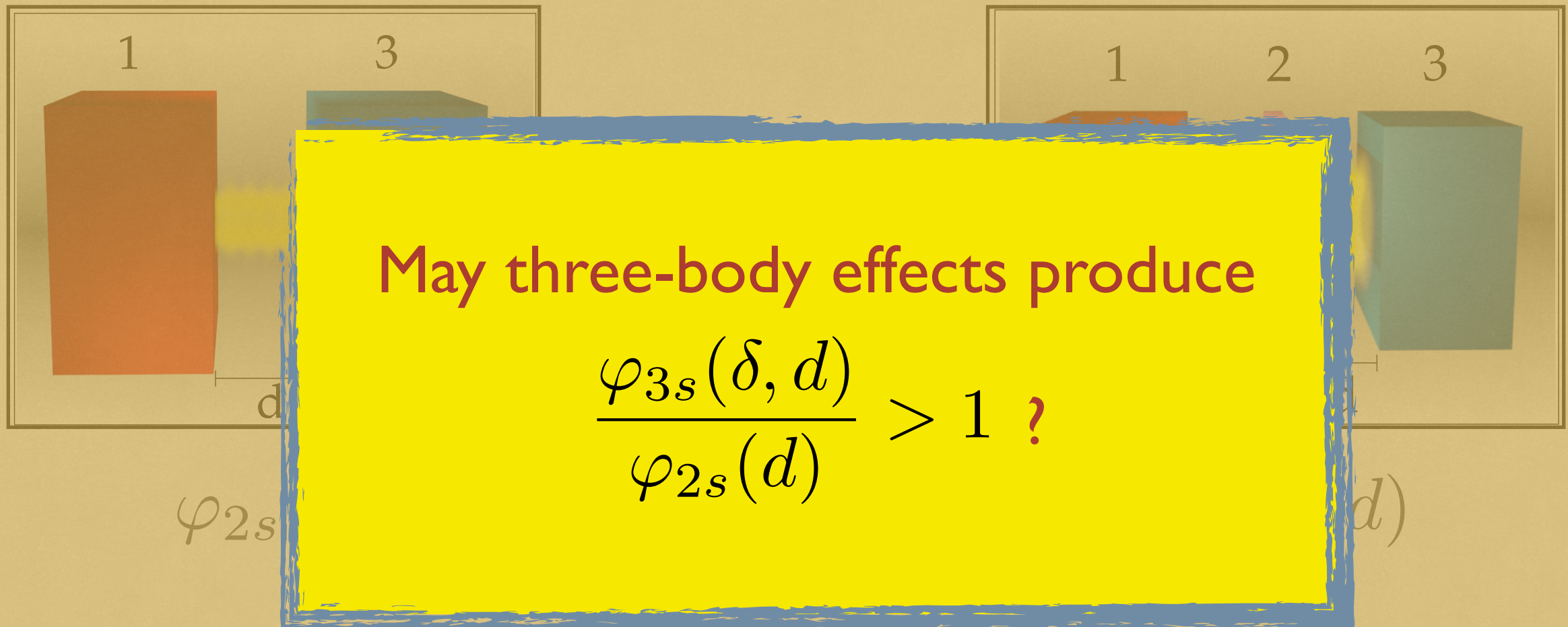
$$\varphi_{3s}(\delta, d)$$

The temperature T_2 is such that the **heat flux on body 2 is zero**

$\Rightarrow$ No additional external thermal source

Amplification of heat transfer

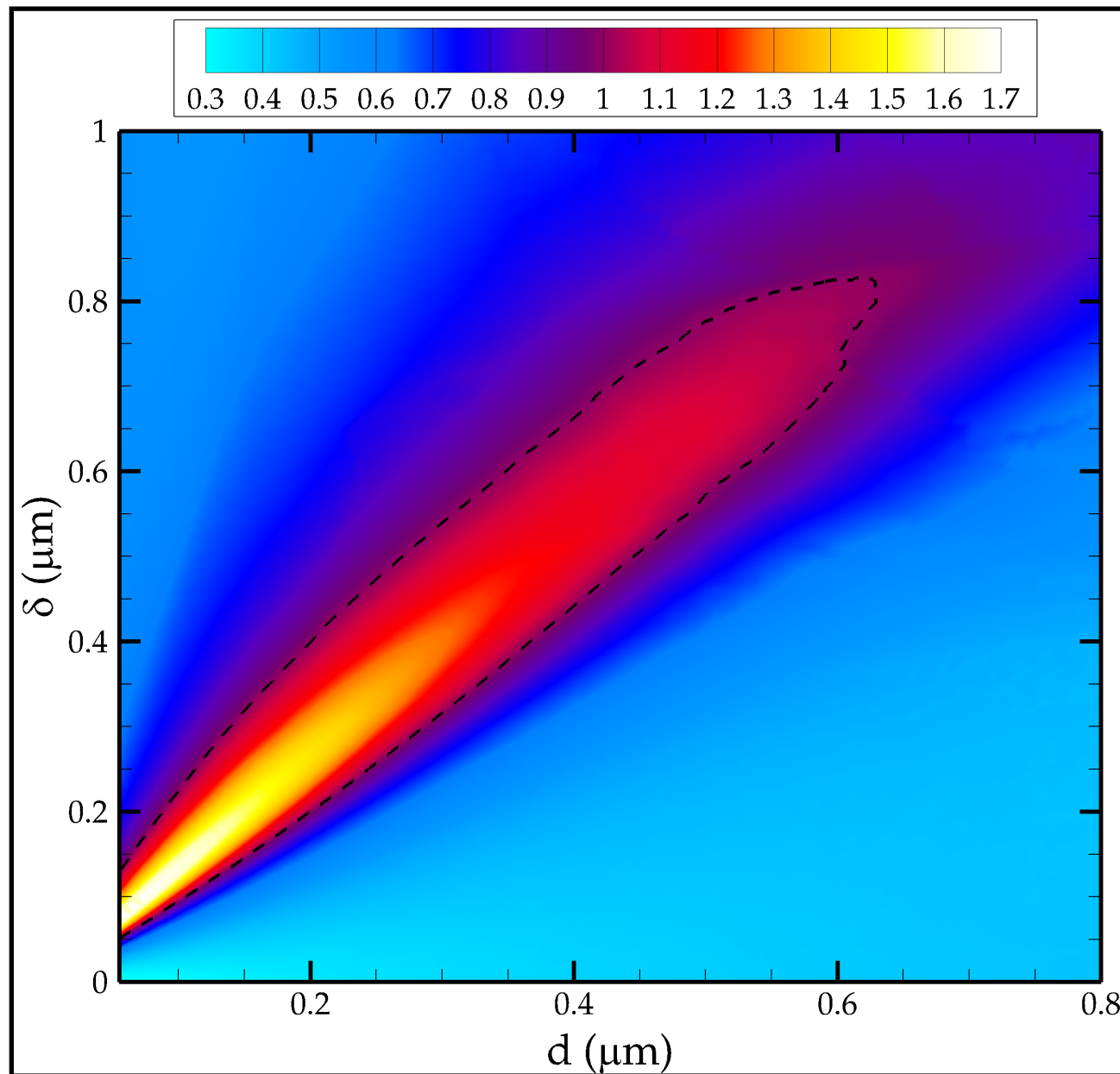
We want to compare



The temperature T_2 is such that the **heat flux on body 2 is zero**

$\Rightarrow$ No additional external thermal source

Amplification of heat transfer



Two SiC slabs
(surface phonon-polariton)

Temperatures

$$T_1 = 400 \text{ K}$$

$$T_2 = 300 \text{ K}$$

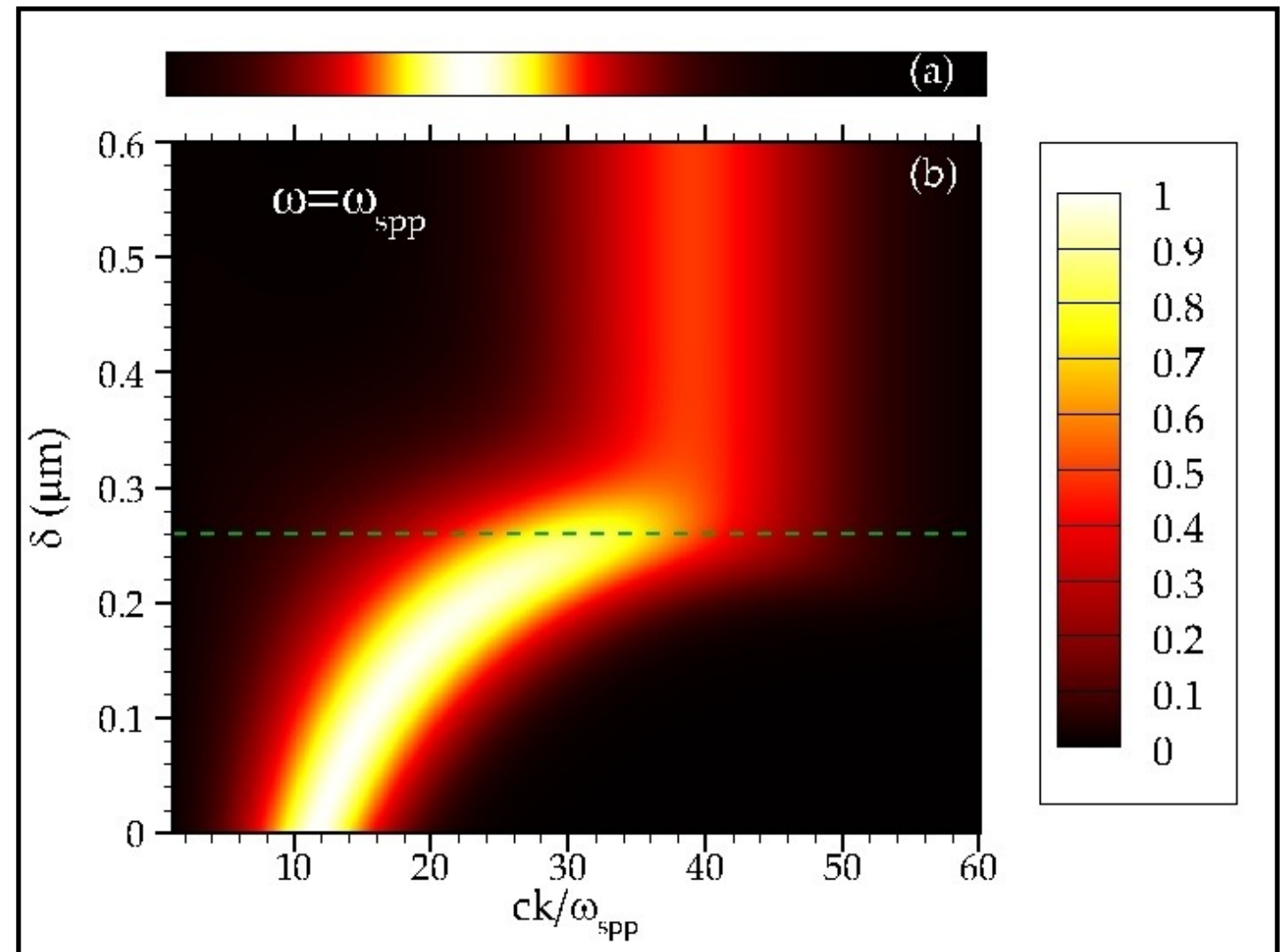
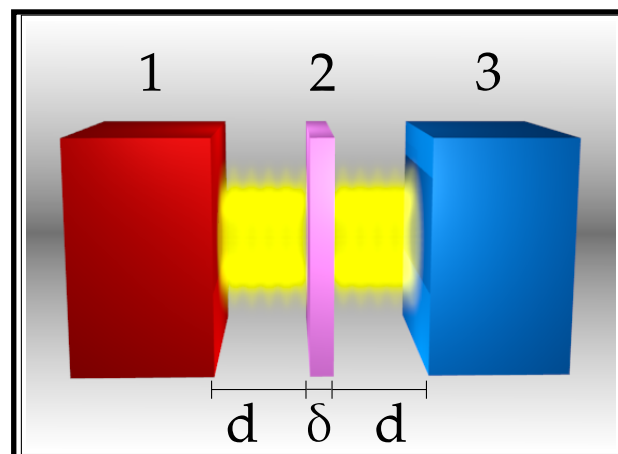
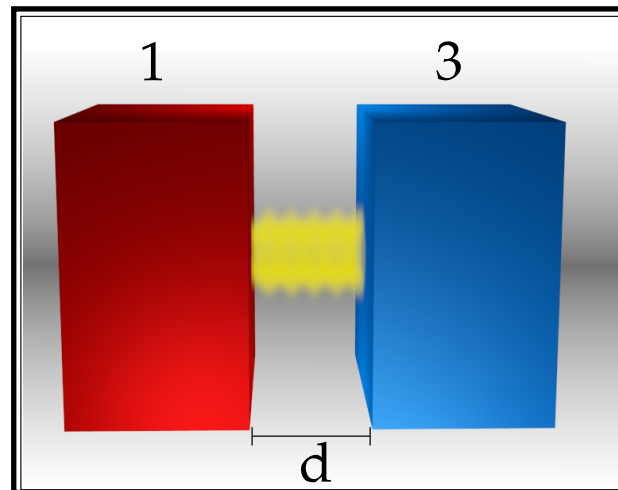
Amplification factor as a function
of distance and thickness

- **Amplification** exists up to distances $d \simeq 600 \text{ nm}$
- For each value of d an optimal value of δ exists

RM, M. Antezza, and P. Ben-Abdallah, PRL **109**, 244302 (2012)

Monochromatic heat flux

Contribution of the modes at the **surface-resonance frequency**



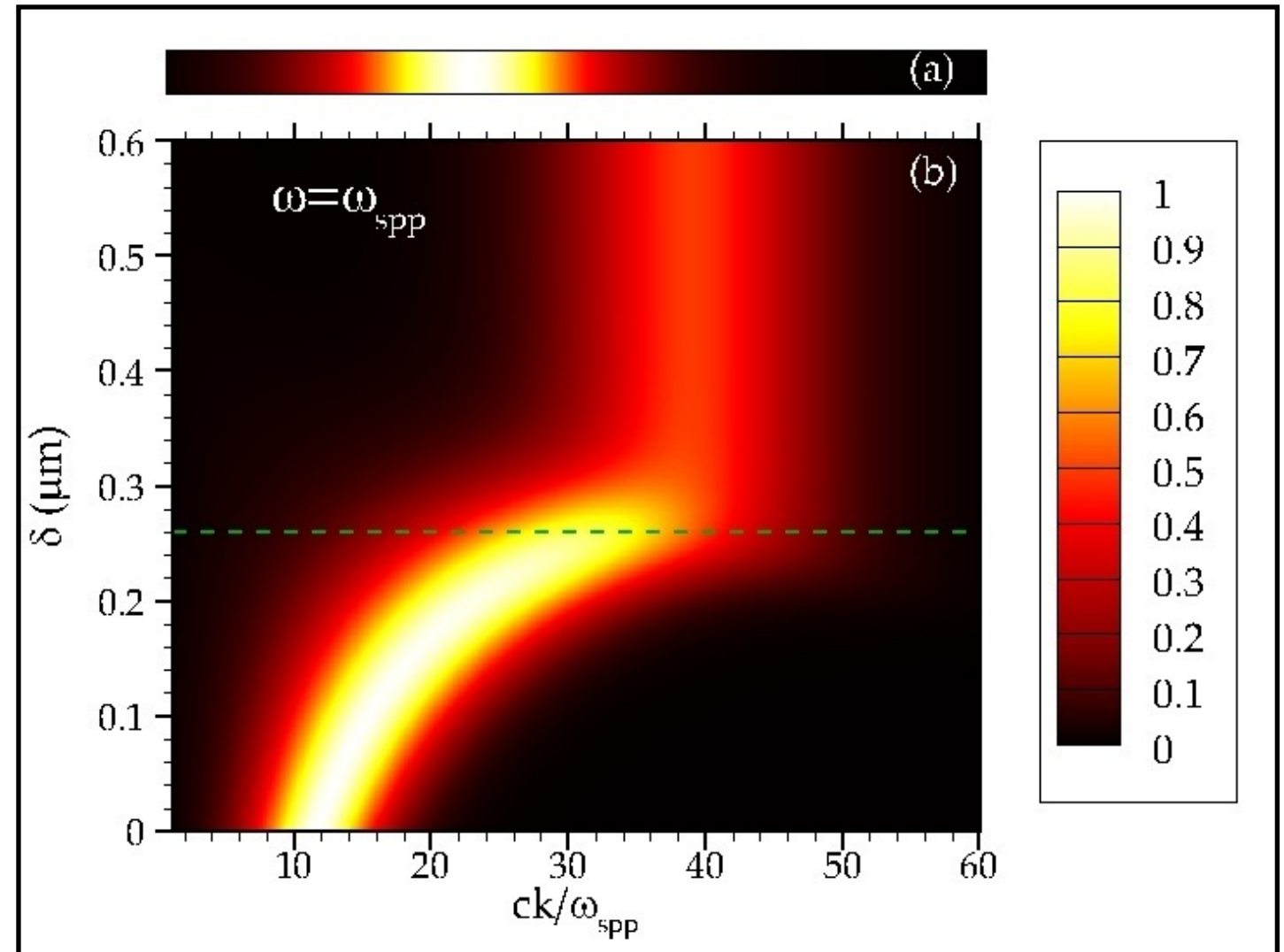
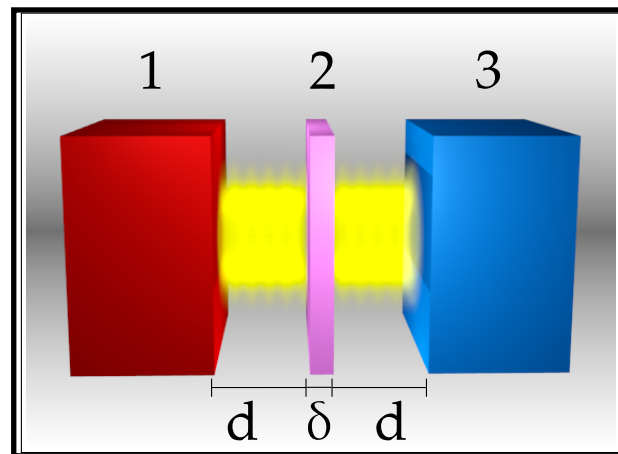
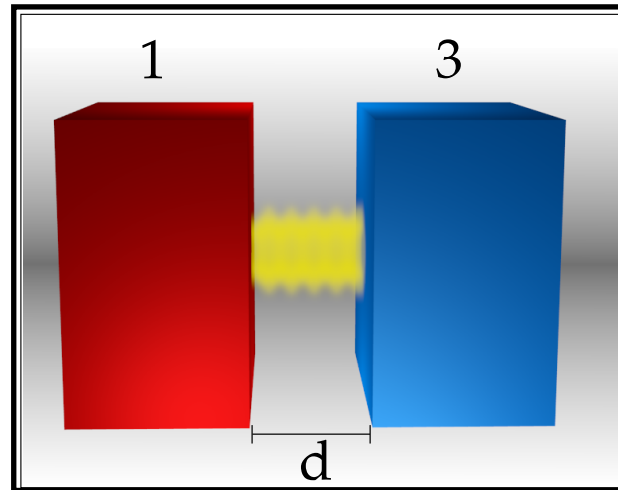
(a) Two slabs, SiC-SiC, distance $d = 200 \text{ nm}$

(b) $\delta = 0 \rightarrow$ Two slabs, distance $2d$

$\delta \gg d \rightarrow$ Two slabs, distance d

Monochromatic heat flux

Contribution of the modes at the **surface-resonance frequency**



The optimal δ corresponds to an **optimized three-body configuration**

The number of participating modes is enhanced

Radiative HT between nanoparticles

Major theoretical simplification

- Each particle described as a **pointlike source**
- Description by means of an electric and/or magnetic **dipole**
- Limitation of the interparticle distance

Modeling tip-surface experiments

Extensive literature on the topic

P. Ben-Abdallah, K. Joulain, J. Drevillon, and C. Le Goff, Phys. Rev. B **77**, 075417 (2008)

P.-O. Chapuis, M. Laroche, S. Volz, and J.-J. Greffet, Appl. Phys. Lett. **92**, 20 (2008)

P. Ben-Abdallah, S.-A. Biehs, and K. Joulain, Phys. Rev. Lett. **107**, 114301 (2011)

V. Yannopapas and N. V. Vitanov, Phys. Rev. Lett. **110**, 044302 (2013)

P. Ben-Abdallah, R. Messina, S.-A. Biehs, M. Tschikin, K. Joulain, and C. Henkel, Phys. Rev. Lett. **111**, 174301 (2013)

M. Nikbakht, J. Appl. Phys. **116**, 094307 (2014)

R. Incardone, T. Emig, and M. Krüger, Europhys. Lett. **106**, 41001 (2014)

J. Dong, J. Zhao, and L. Liu, Phys. Rev. B **95**, 125411 (2017)

K. Asheichyk, B. Müller, and M. Krüger, Phys. Rev. B **96**, 155402 (2017)

and others...

Discrete Dipole Approximation

- Macroscopic body described as a set of dipoles

E. M. Purcell and C. R. Pennypacker, Astrophys. J. **186**, 705 (1973)

Thermoplasmonic applications

- Nanochemistry, photothermal therapy

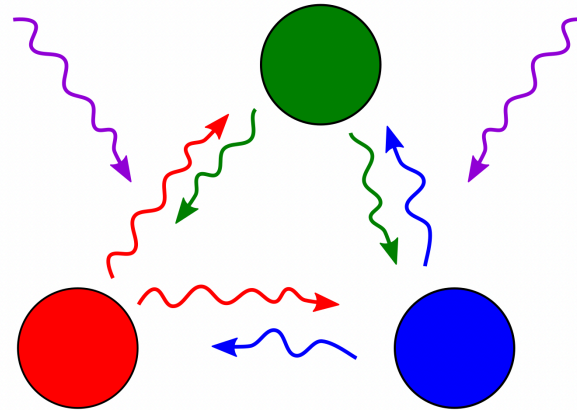
G. Baffou and R. Quidant, Laser Photon. Rev. **7**, 171 (2013)

Self-consistent formalism

Electric field

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{(b)}(\mathbf{r}) + \mathbf{E}^{(ind)}(\mathbf{r})$$

$$\mathbf{E}^{(ind)}(\mathbf{r}) = \frac{k^2}{\varepsilon_0} \sum_i \mathbb{G}(\mathbf{r}, \mathbf{r}_i) \mathbf{p}_i$$



Dipole moments

$$\mathbf{p}_i = \mathbf{p}_i^{(fl)} + \mathbf{p}_i^{(ind)}$$

$$\mathbf{p}_i^{(ind)} = \varepsilon_0 \alpha_i \left(\mathbf{E}_i^{(b)} + \frac{k^2}{\varepsilon_0} \sum_{j \neq i} \mathbb{G}(\mathbf{r}_i, \mathbf{r}_j) \mathbf{p}_j \right)$$

- **Fluctuating + induced** contributions
- **Green's function** (geometry of the system)
- System of equations: total field and dipole moments as a function of the fluctuating terms

Absorbed power

$$\begin{aligned} \mathcal{P}_i^{(abs)}(t, T_1, \dots, T_N, T_b) &= \left\langle \frac{d\mathbf{p}_i(t)}{dt} \cdot \mathbf{E}(\mathbf{r}_i, t) \right\rangle \\ &= \int_0^{+\infty} \frac{d\omega}{2\pi} \hbar \omega \left[\sum_{j \neq i} \frac{4\chi_i \chi_j}{|\alpha_i|^2} n_{ji}(\omega) \text{Tr} \left(\mathbb{T}_{ij}^{-1} \mathbb{T}_{ji}^{-1\dagger} \right) + \frac{4k^2 \chi_i}{|\alpha_i|^2} n_{bi}(\omega) \sum_{jk} \alpha_j \alpha_k^* \text{Tr} \left(\mathbb{T}_{ij}^{-1} \text{Im}(\mathbb{G}_{jk}^{(0)}) \mathbb{T}_{ki}^{-1\dagger} \right) \right] \end{aligned}$$

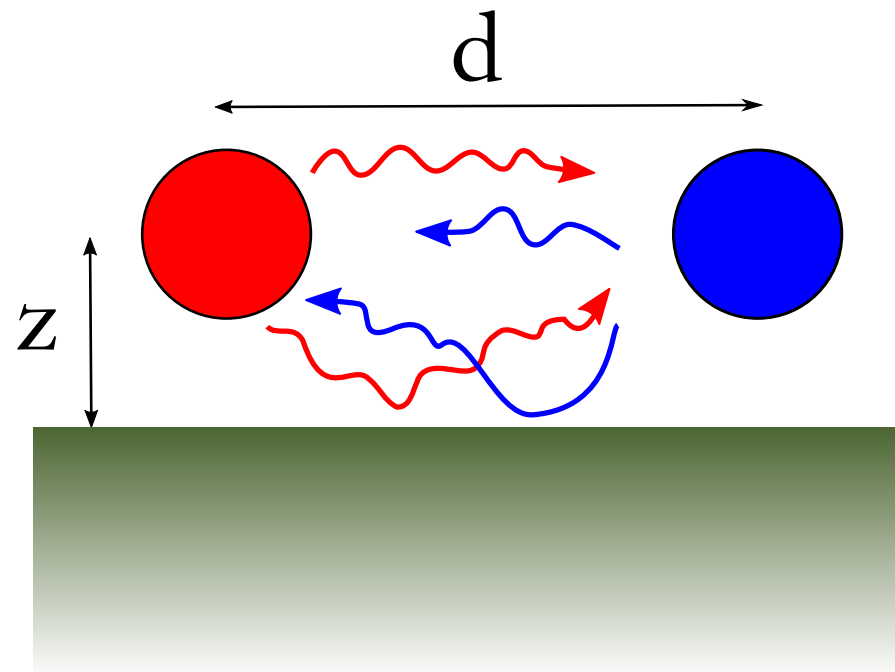
Exchange between bodies

Trace expression

Exchange with external field

Susceptibilities of the nanoparticles

Two nanoparticles above a substrate



- **Two nanospheres** at different temperatures
- **Two distances**: interparticle and particle-surface
- Limitation of the interparticle distance $R \ll d, z, \lambda$
- $R = 5 \text{ nm}$

Conductance

$$G = 4 \int_0^{+\infty} \frac{d\omega}{2\pi} \hbar \omega k_0^4 n'(\omega, T) \chi^2 \text{Tr}(\mathbb{G} \mathbb{G}^\dagger)$$

Green functions

$$\mathbb{G} = \mathbb{G}^{(0)} + \mathbb{G}^{(\text{SC})}$$

Vacuum + scattering contribution $\longrightarrow$ Ex.: EE term

T - derivative of BE distribution

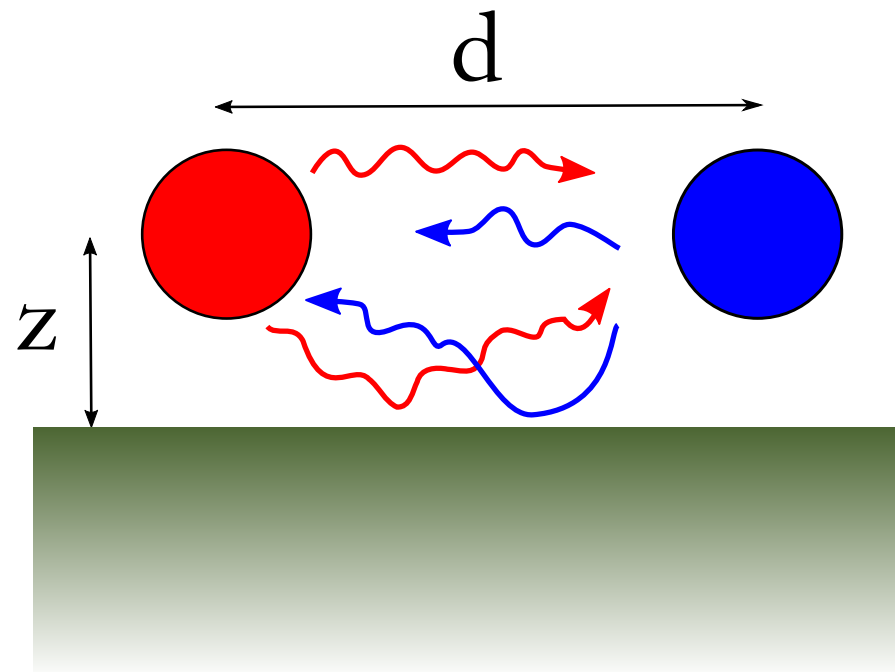
Polarizabilities

$$\alpha_E^{(0)}(\omega) = 4\pi R^3 \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}$$

$$\alpha_H^{(0)}(\omega) = \frac{2\pi}{15} R^3 \left(\frac{\omega R}{c} \right)^2 [\epsilon(\omega) - 1]$$

$$\mathbb{G}_{EE}^{(\text{SC})} = \int_0^{+\infty} \frac{dk}{2\pi} \frac{i k e^{2ik_z z}}{2k_0^2 k_z} (r_s \mathbb{S} + r_p \mathbb{P})$$

Some theoretical remarks



Near and far field

- The **interparticle distance** regulates the flux amplification even in vacuum
- The **particle-surface distance** regulates the participation of the surface mode

Three contributions to conductance

$$\text{Tr}(\mathbb{G}\mathbb{G}^\dagger) = \text{Tr}(\mathbb{G}^{(0)}\mathbb{G}^{(0)\dagger}) + \text{Tr}(\mathbb{G}^{(\text{SC})}\mathbb{G}^{(\text{SC})\dagger}) + 2\text{Re}[\text{Tr}(\mathbb{G}^{(0)}\mathbb{G}^{(\text{SC})\dagger})]$$

Total conductance

$$G = G^{(0,0)} + G^{(\text{SC},\text{SC})} + G^{(0,\text{SC})}$$

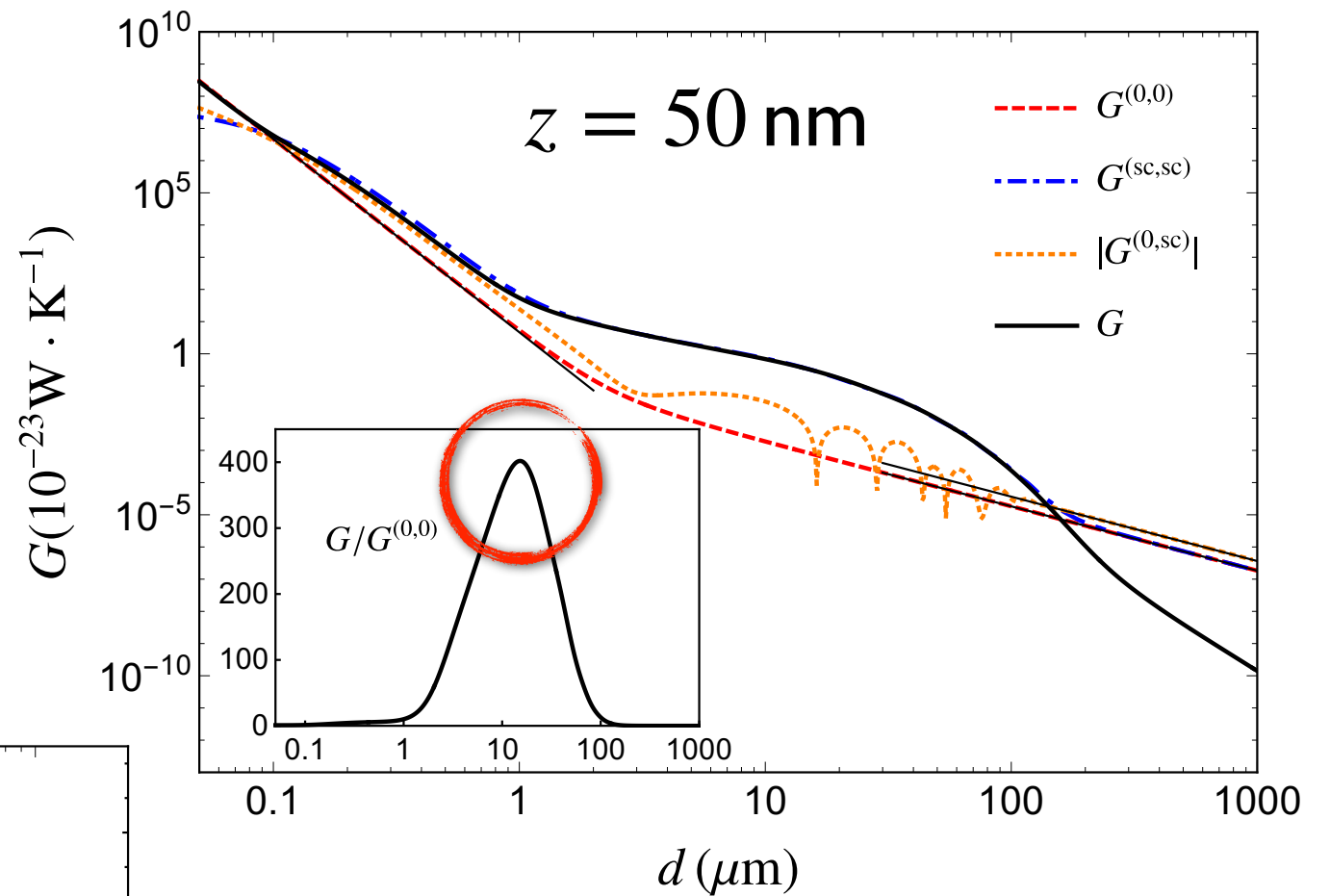
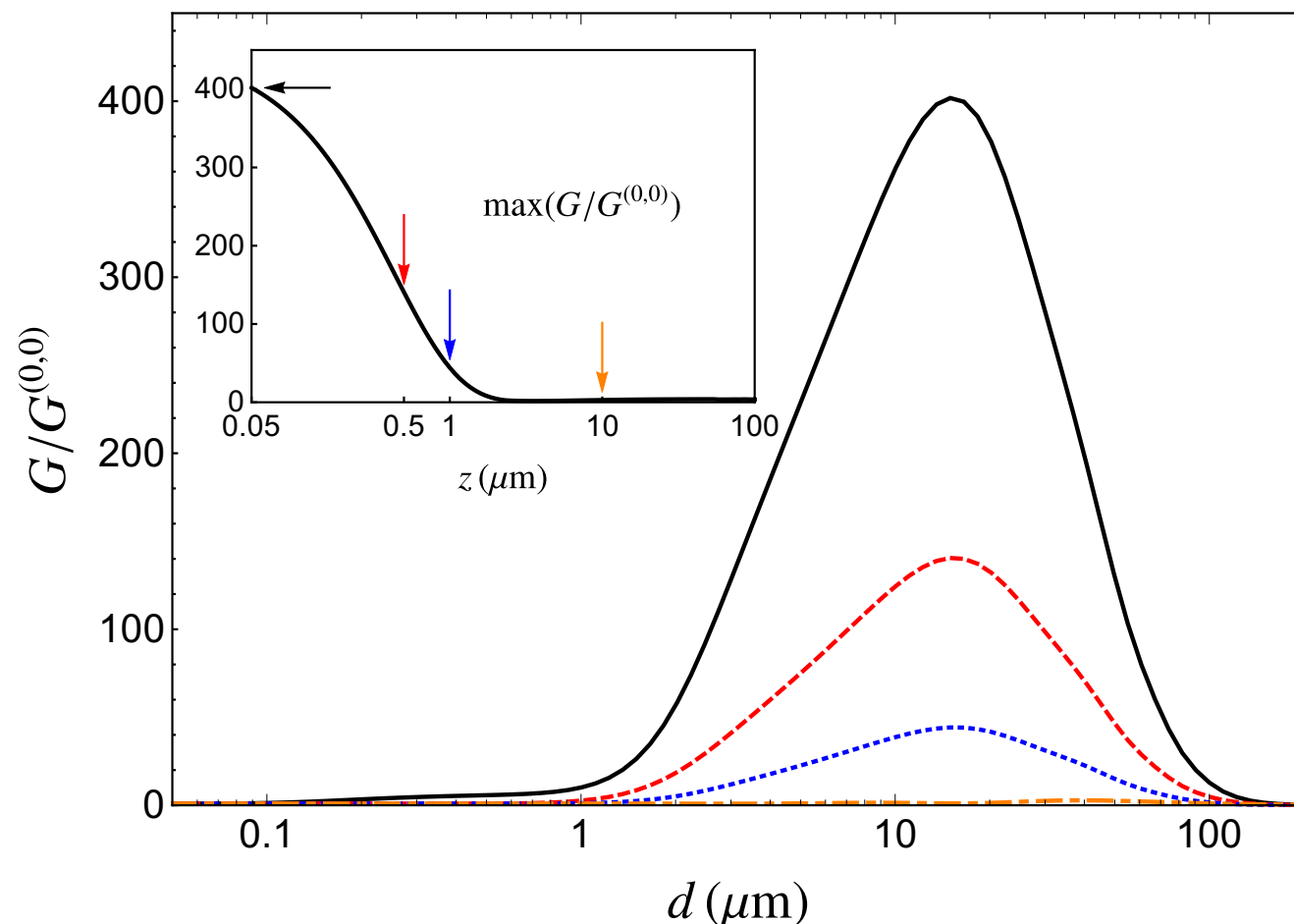
$\swarrow$ $\downarrow$ $\searrow$
 vacuum + scattering + mixed

Amplification

$$G/G^{(0,0)}$$

SiC particles on SiC substrate

- **Conductance** as a function of the interparticle distance for $z = 50$ nm
- **Amplification** of two orders of magnitude on a large range of distances
- **Modified** long-distance **power-law** behavior

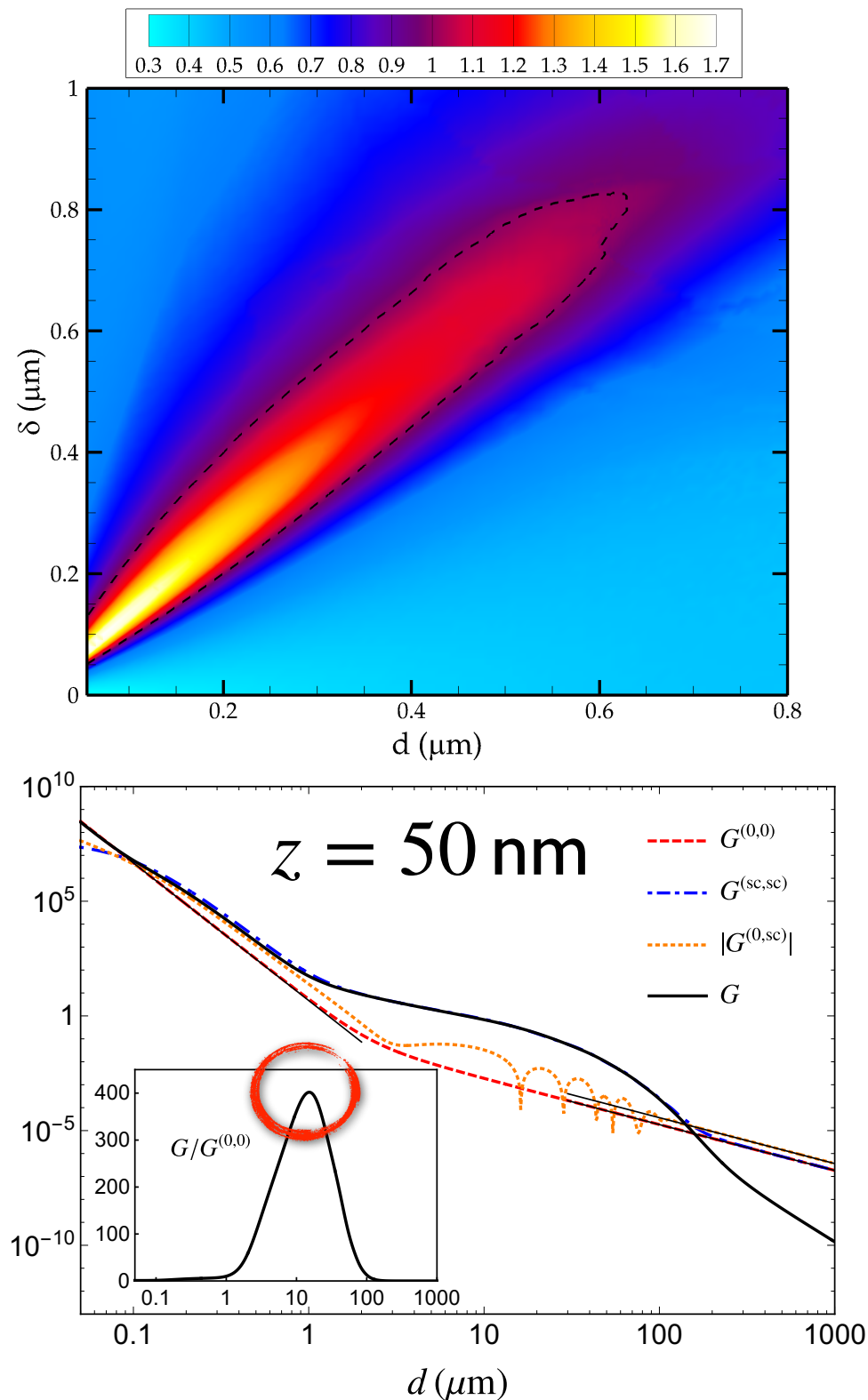


Dependence on z:

- Signature of the participation of the surface mode
- Decay length compatible with phonon-polariton decay length

J. Dong, J. Zhao, and L. Liu, PRB **97**, 075422 (2018)
 RM, S.-A. Biehs, and P. Ben-Abdallah, PRB **97**, 165437 (2018)

Many-body effects



- Radiative HT between two bodies can be strongly amplified by exploiting the **presence of a third body**
- The amplification can go up to **two orders of magnitude**
- The number of participating modes and the spectral features are modified

RM, M. Antezza, and P. Ben-Abdallah, PRL **109**, 244302 (2012)

J. Dong, J. Zhao, and L. Liu, PRB **97**, 075422 (2018)

RM, S.-A. Biehs, and P. Ben-Abdallah, PRB **97**, 165437 (2018)

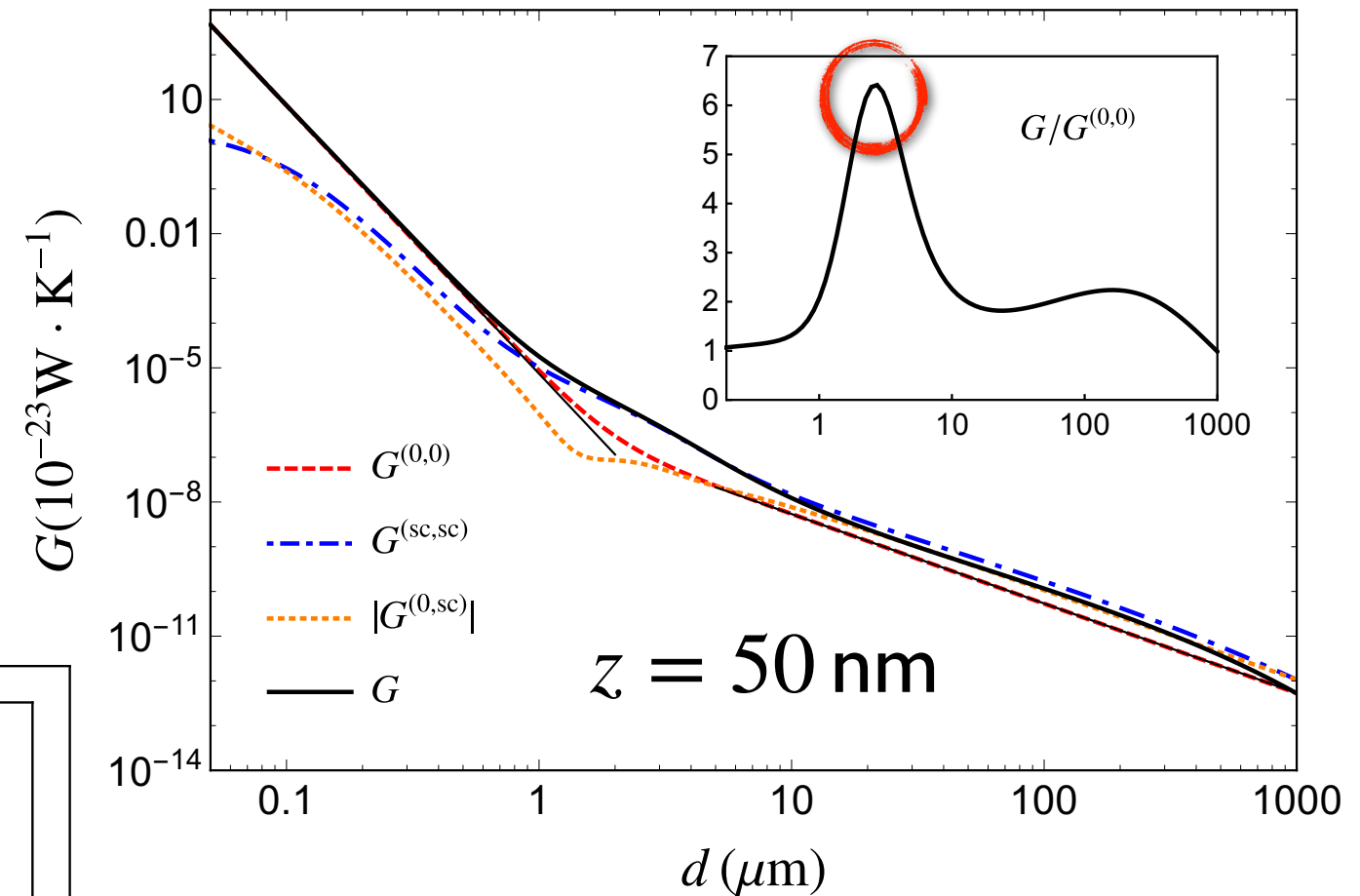
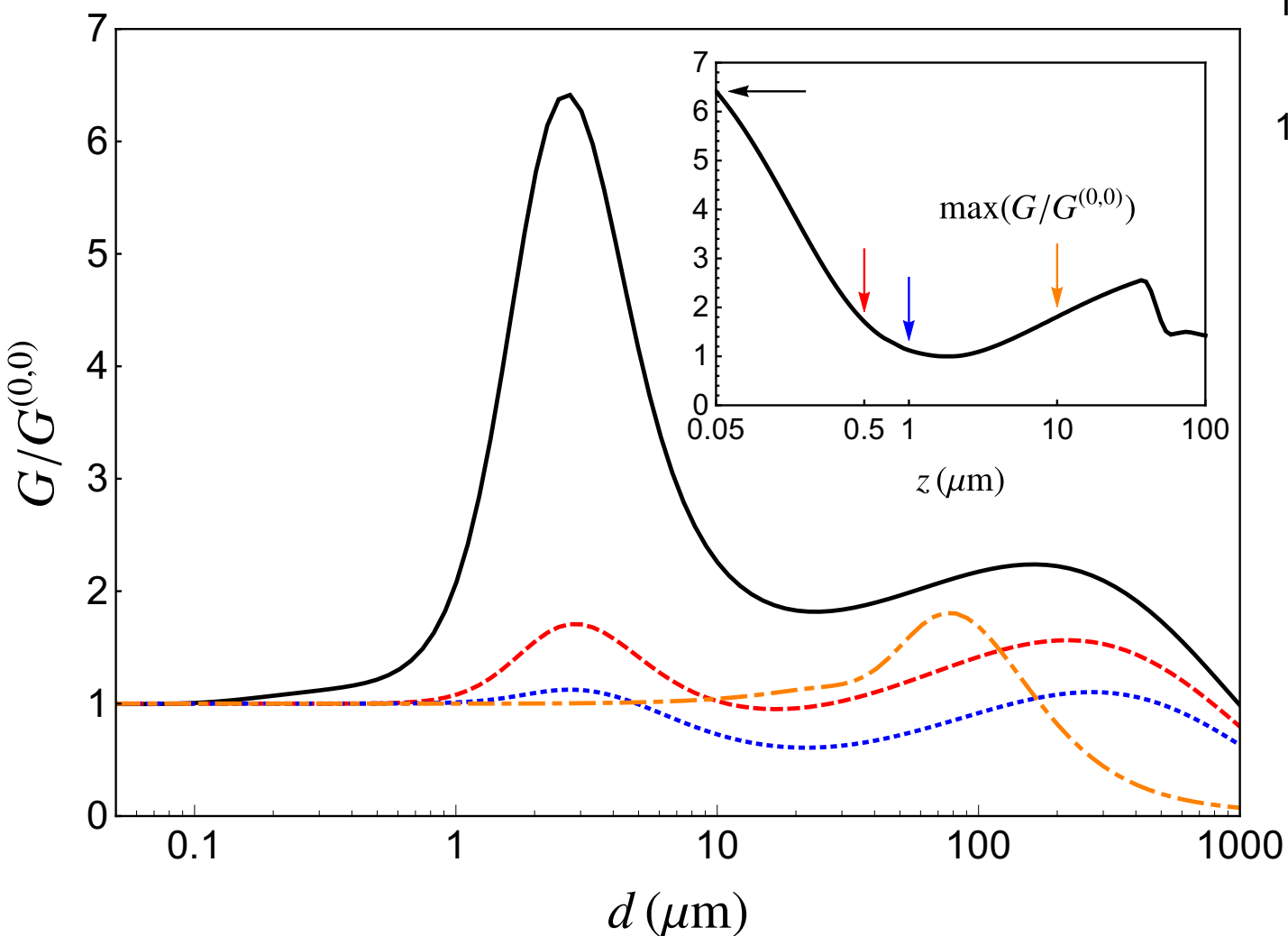
RM et al., Phys. Rev. B **88**, 104307 (2013)

Supplementary slides

Two gold particles on substrate

Au particles on SiC substrate

- Much **smaller values** of the conductance
- **Conductance** as a function of the interparticle distance for $z = 50$ nm
- Much smaller **amplification**



Dependence on z :

- Signature of the participation of the surface mode
- Decay length compatible with phonon-polariton decay length

Spectral analysis

Two kind of surface modes

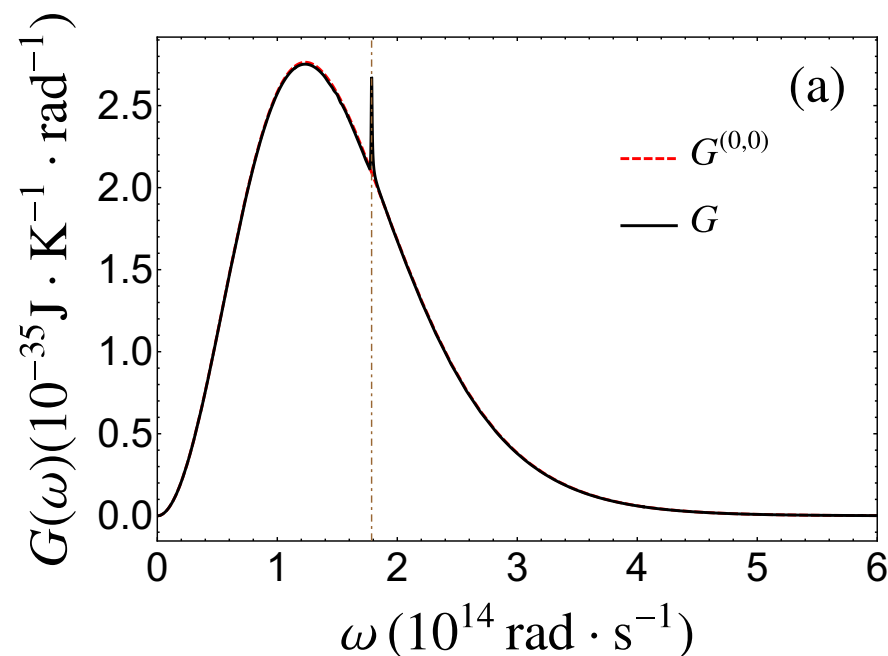
Planar SiC-vacuum interface

$$\varepsilon(\omega) + 1 = 0$$

$$\omega_{\text{pl}} = 1.786 \times 10^{14} \text{ rad/s}$$

Expected to contribute for small z

$$d = 50 \text{ nm}$$



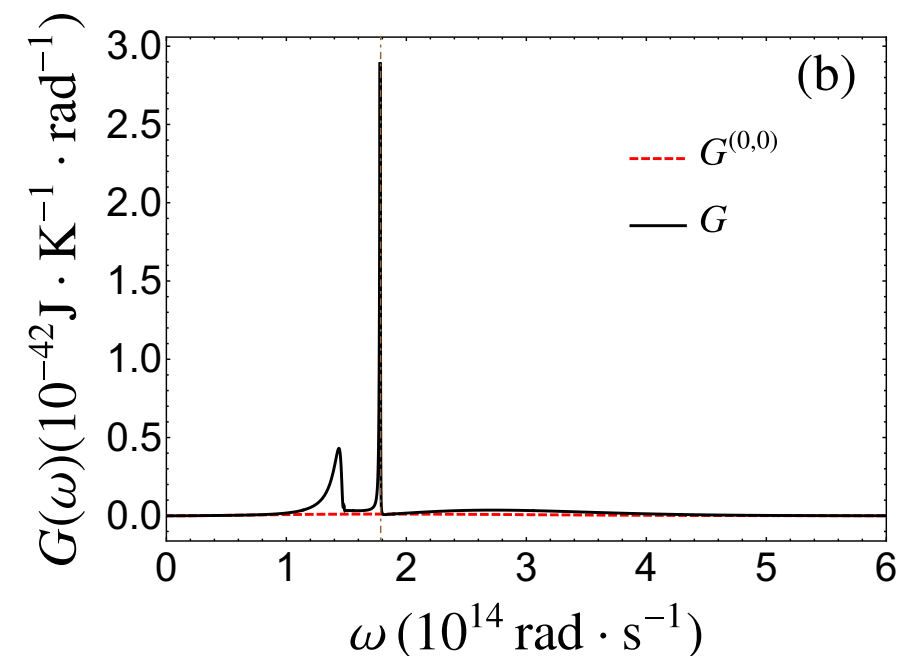
Spherical Au-vacuum interface

$$\varepsilon(\omega) + 2 = 0$$

$$\omega_{\text{np}} = 7.9 \times 10^{15} \text{ rad/s}$$

Ultraviolet range, thermally not excited

$$d = 2 \mu\text{m}$$



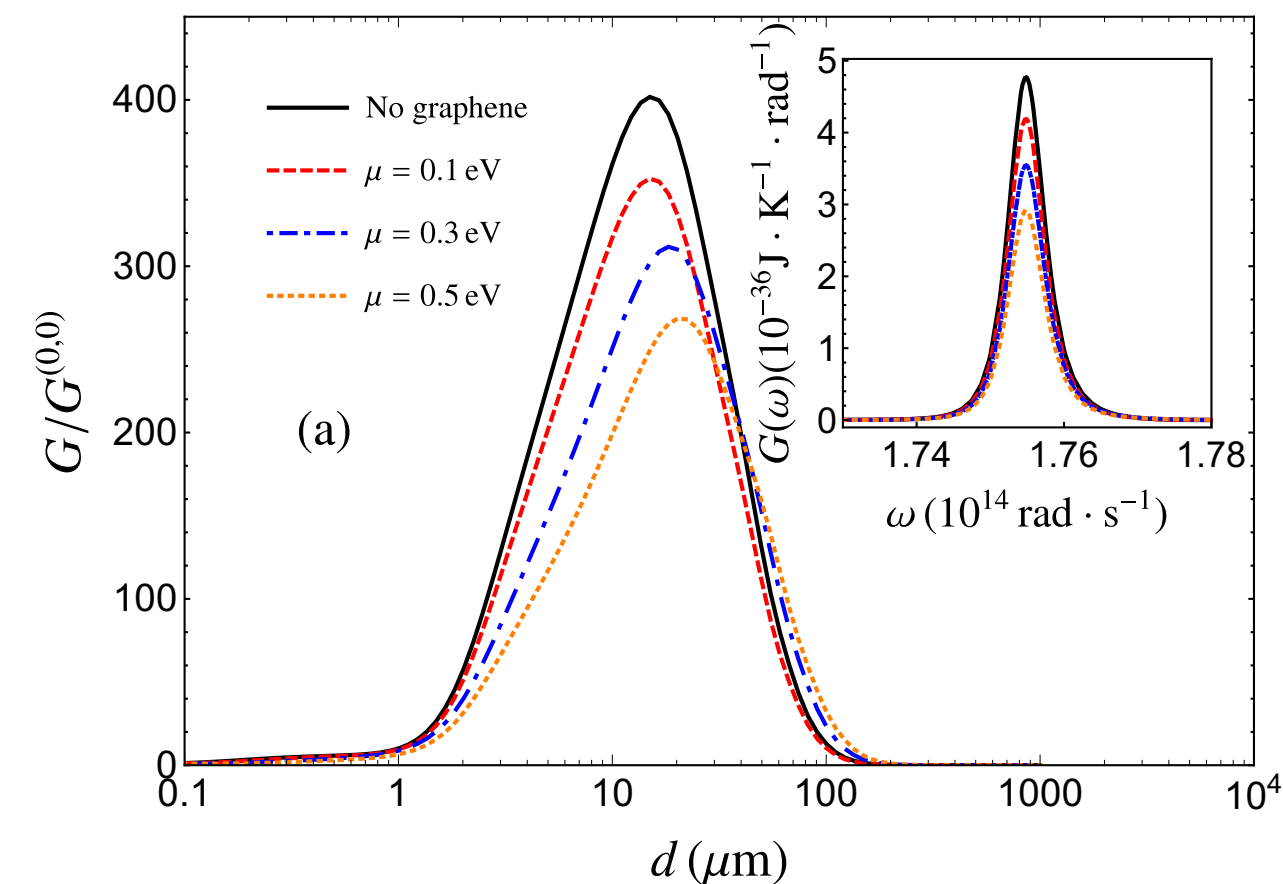
Graphene sheet on the substrate

Recently, the use of **graphene** has been theoretically proposed to increase the near-field RHT by **reducing the mismatch** between the surface modes of **different materials**

R. Messina and P. Ben-Abdallah, Sci. Rep. **3**, 1383 (2013)

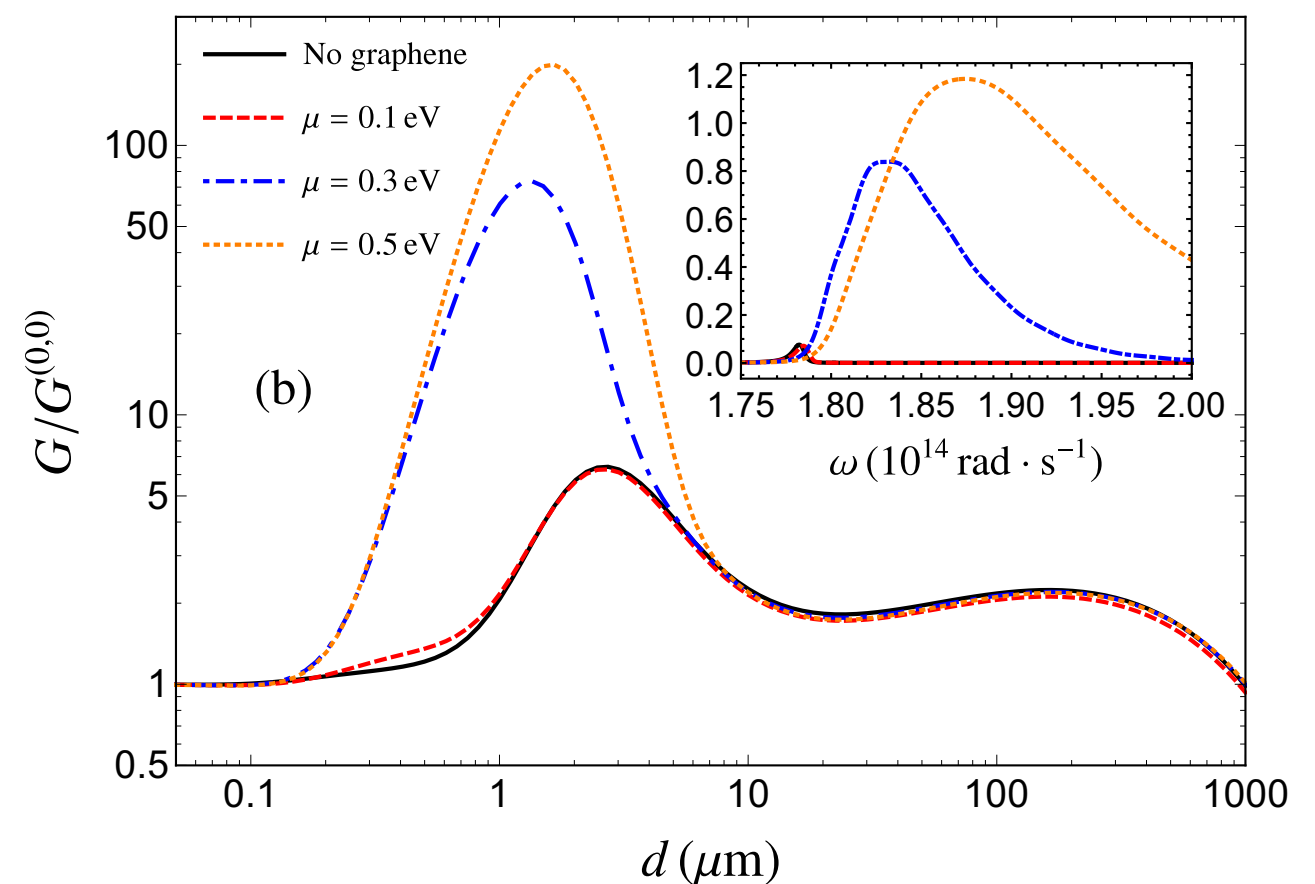
R. Messina, P. Ben-Abdallah, B. Guizal, and M. Antezza, Phys. Rev. B **96**, 045402 (2017)

SiC particles



Graphene is not beneficial

Au particles

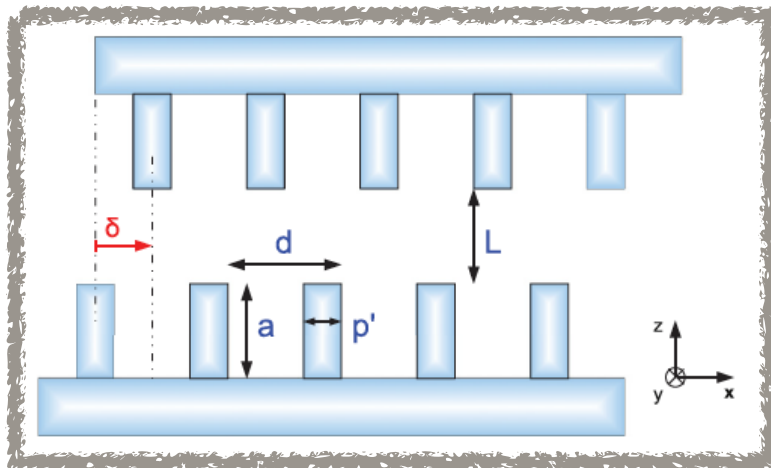


Huge increase of conductance amplification

Radiative heat transfer between gratings

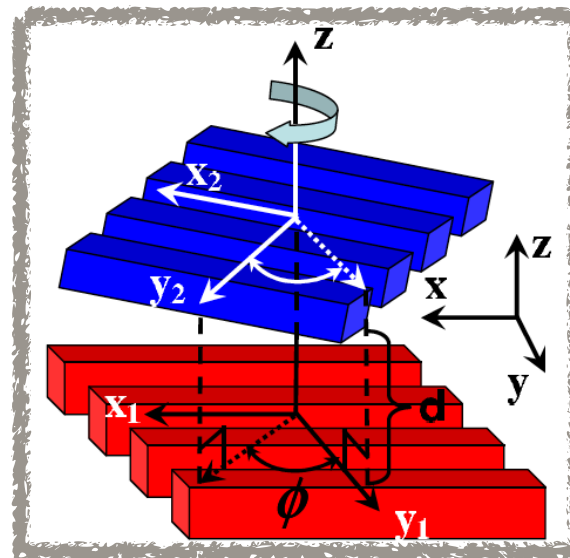
Radiative heat transfer and gratings

S-matrix approach



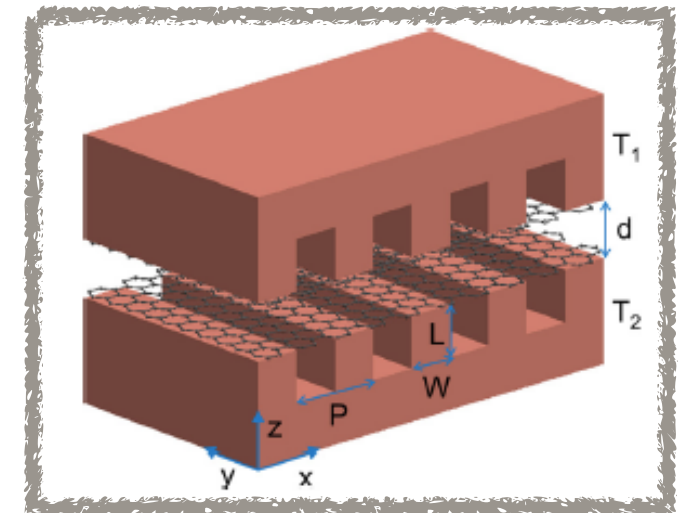
J. Lussange et al., PRA **86**, 062502 (2012);
PRB **86**, 085432 (2012)
R. Guérout et al., PRB **85**, 180301(R) (2012)

Effective theory



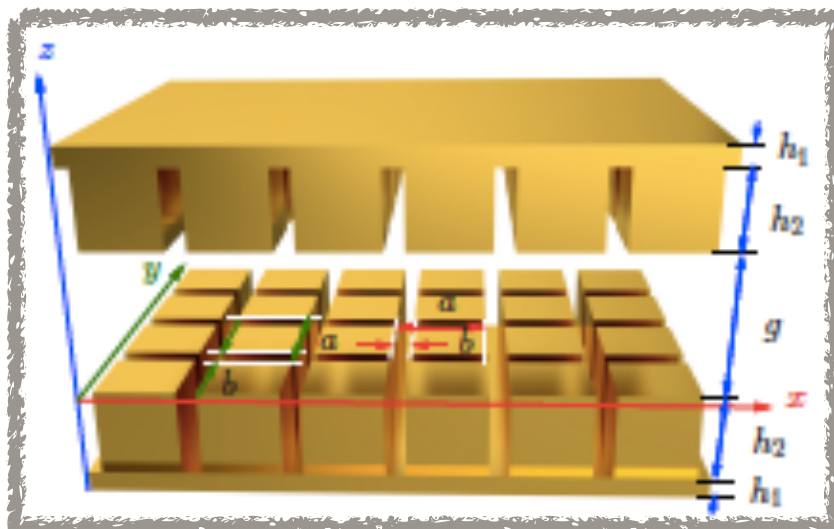
S.-A. Biehs et al., APL **98**, 243102 (2011)

Role of graphene



X. L. Liu et al., APL **104**, 251911 (2014)

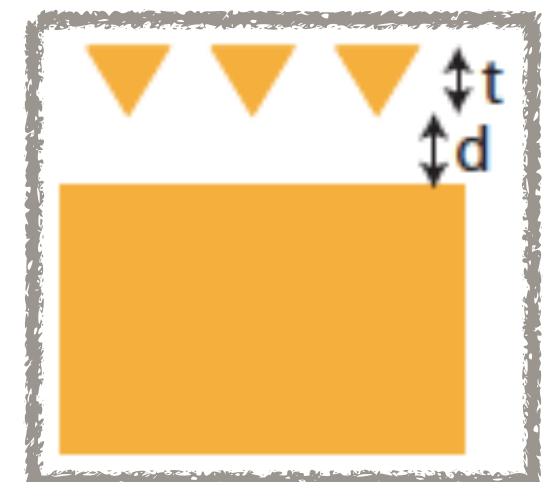
1D and 2D gratings



J. Dai et al., PRB **92**, 035419 (2015);
PRB **93**, 155403 (2016); PRB **94**, 125431 (2016)

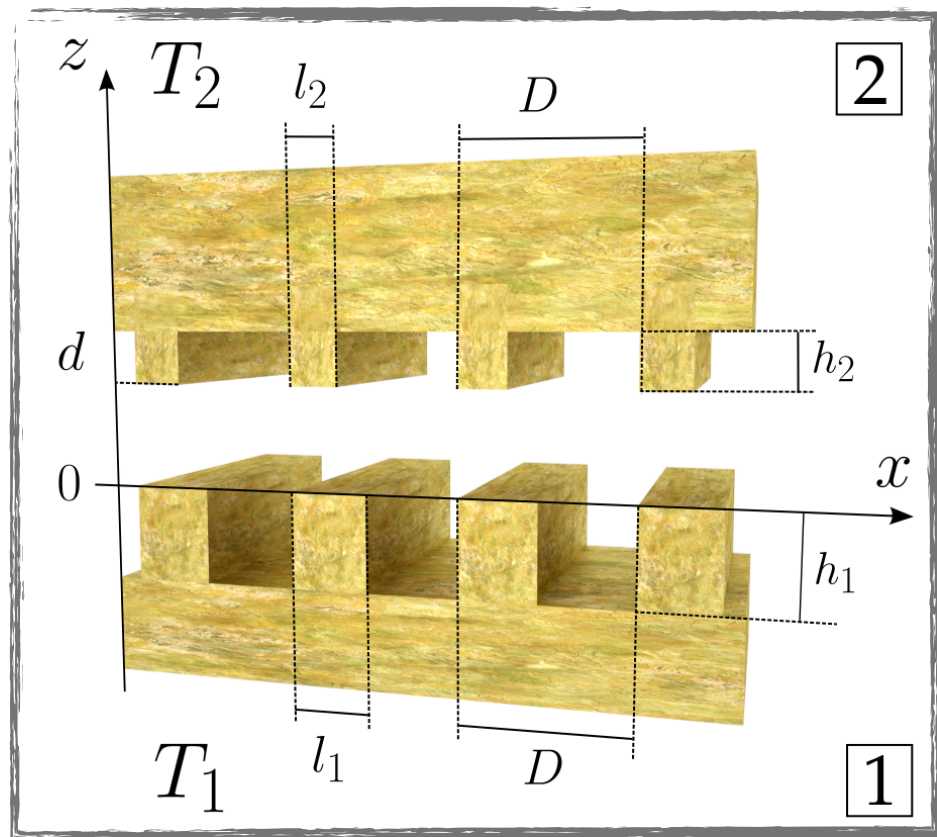
- Manipulation of **spectral** properties
- New field **modes**
- **Enhancement** or **suppression** of the effect

Effect of shape



H. Chalabi et al., PRB **91**, 014302 (2015);
PRB **91**, 174304 (2015); PRB **94**, 094307 (2016)

The physical problem



Grating parameters

- Periodic in x direction
- Invariant in y direction
- Period D
- Filling factor $f_i = l_i/D$
- Thickness δ_i
- Corrugation height h_i

Parameters

Substrate thicknesses

$$\delta_1 = \delta_2 = \infty$$

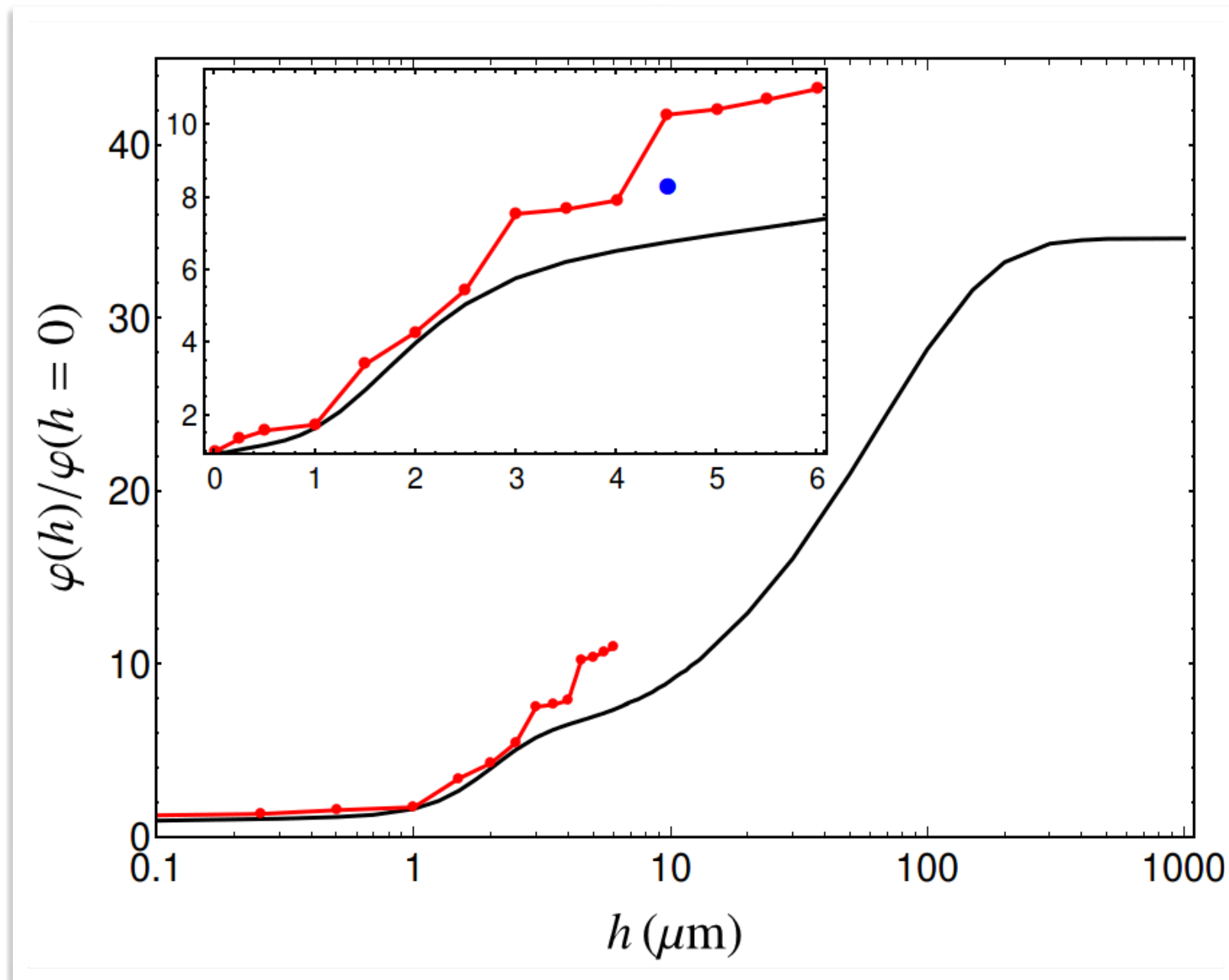
$$D_1 = D_2 = 1 \mu\text{m}$$

$$d = 1 \mu\text{m}$$

Flux amplification

$$\varphi(h)/\varphi(0)$$

Amplification of heat transfer



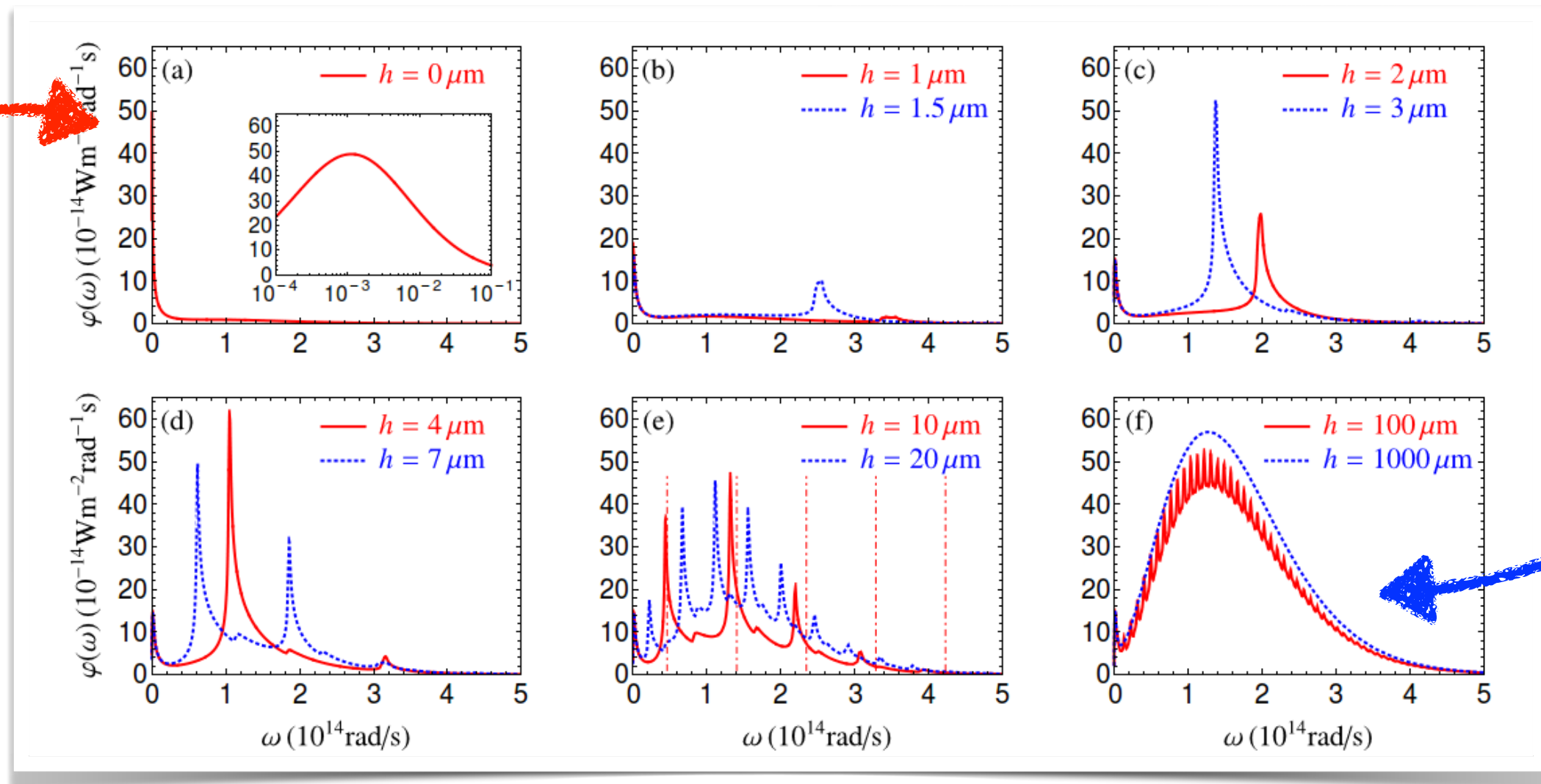
Amplification of more than one order of magnitude!

R. Guérout et al., PRB **85**, 180301(R) (2012)

RM, A. Noto, B. Guizal, and M. Antezza, PRA **95**, 125404 (2017)

Spectral analysis vs grating depth

Planar slab



Asymptotic profile

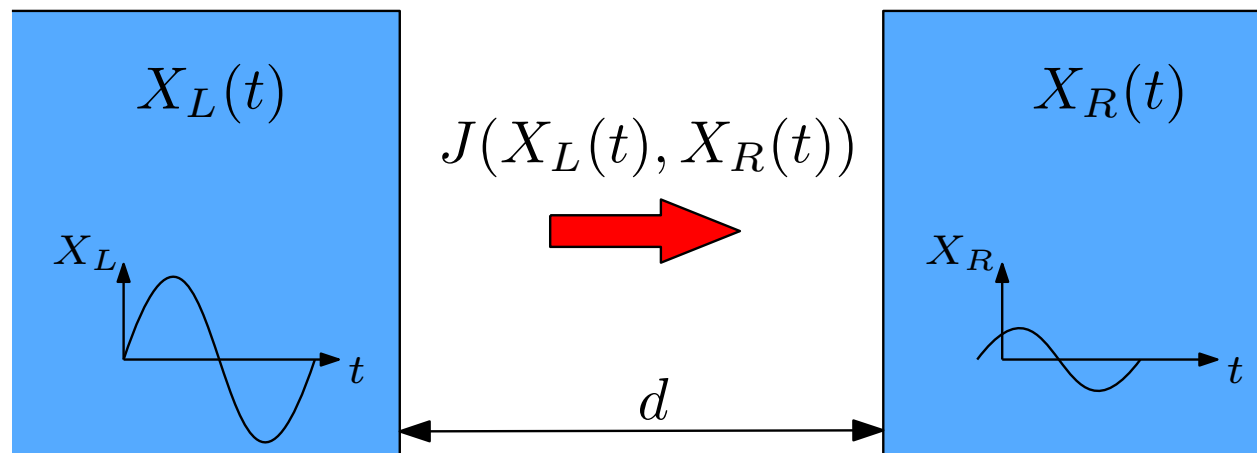
- Appearance of **new modes**
- **Number** and **position** of modes strong function of h
- **Spoof plasmons**, approximately described by the relation (perf. conductors)

$$\omega_n = (2n + 1)\pi c / 2h$$

J. B. Pendry et al., Science **305**, 847 (2004); F. J. García-Vidal et al., J. Opt.A: Pure Appl. Opt. **7**, S97 (2005)

Shuttling and pumping effects

Heat shuttling



- Energy exchange between bodies having **time-modulated parameters**
- Active dynamic management of heat flux at the nanoscale

$$X_L(t), X_R(t) \longrightarrow \text{Average difference} \quad \Delta \bar{X} = \frac{1}{\tau} \int_0^\tau \Delta X(t) dt = x_L - x_R$$

Flux for averaged variables

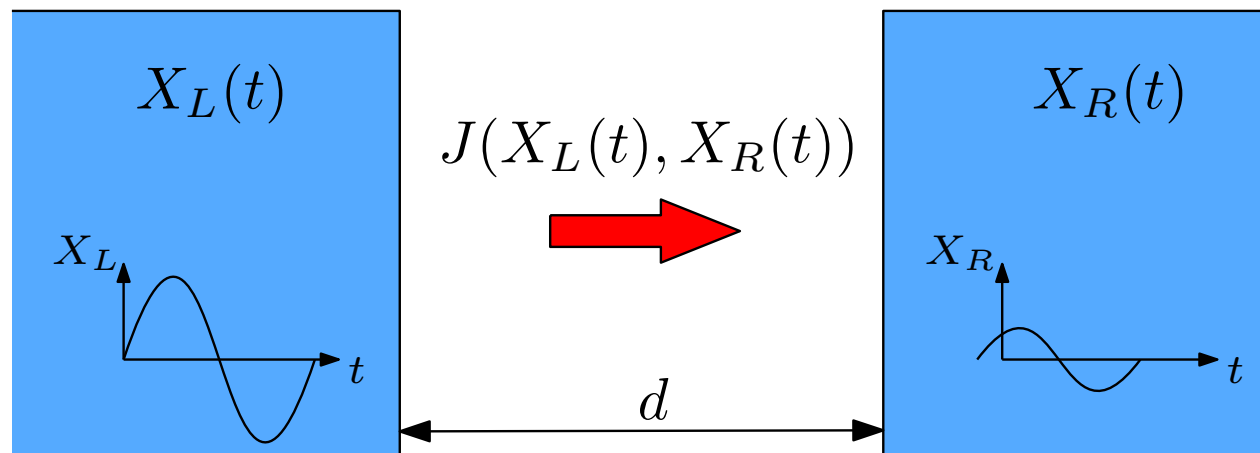
+

Supplementary shuttling flux due to **nonlinearity** which can reduce or increase the flux

Already studied for conduction

- N.-B. Li, P. Hanggi, and B. Li, Europhys. Lett. **84**, 40009 (2008)
 N.-B. Li, F. Zhan, P. Hanggi, and B. Li, Phys. Rev. E **80**, 011125 (2009)
 J. Ren and B. Li, Phys. Rev. E **81**, 021111 (2010).
 J. Ren, P. Hanggi, and B. Li, Phys. Rev. Lett. **104**, 170601 (2010)
 J. Ren, S. Liu, and B. Li, Phys. Rev. Lett. **108**, 210603 (2012)

Definitions and heat flux



Choice of time dependence

$$X_L(t) = x_L + \delta_L \sin(\Omega t)$$

$$X_R(t) = x_R + \delta_R \sin(\Omega' t + \phi)$$

Temperature or chemical potential

Flux

$$J(X_L(t), X_R(t)) = \int_0^\infty \frac{d\omega}{2\pi} \Theta_{LR} K_{LR}$$

Effective number of modes

$$K_{LR}(\omega, X_L(t), X_R(t)) > 0$$

(e.g. Landauer approach)

$$\Theta_{LR}(\omega, X_L(t), X_R(t)) = \Theta_L(\omega, X_L(t)) - \Theta_R(\omega, X_R(t))$$

$$\Theta_i(\omega, X_i) = \begin{cases} \hbar\omega [e^{\hbar\omega/(k_B X_i)} - 1]^{-1}, & X_i = T_i \\ \hbar\omega [e^{(\hbar\omega - X_i)/(k_B T_i)} - 1]^{-1}, & X_i = \mu_i \end{cases}$$

Perturbative analysis with respect
to the oscillation amplitudes

Perturbative expansion

- Only $X_L(t)$ is allowed to vary
- Expansion with respect to δ_L
- We assume $x_L = x_R = x_0$ thus we have a vanishing zero-order average flux
- We compute the **time-averaged flux** $\bar{J} = \frac{1}{\tau} \int_0^\tau J(t) dt$

$$\bar{J} \simeq \frac{\delta_L^2}{4} \int_0^\infty \frac{d\omega}{2\pi} (\Theta_L'' K_{LR} + 2\Theta_L' K_{LR}')$$

Case 1: optical properties do not depend on $X_L(t)$

$$\bar{J} \simeq \frac{\delta_L^2}{4} \int_0^\infty \frac{d\omega}{2\pi} \Theta_L'' K_{LR} \quad \text{where} \quad \Theta_L'' > 0$$

The shuttling flux is **always positive**

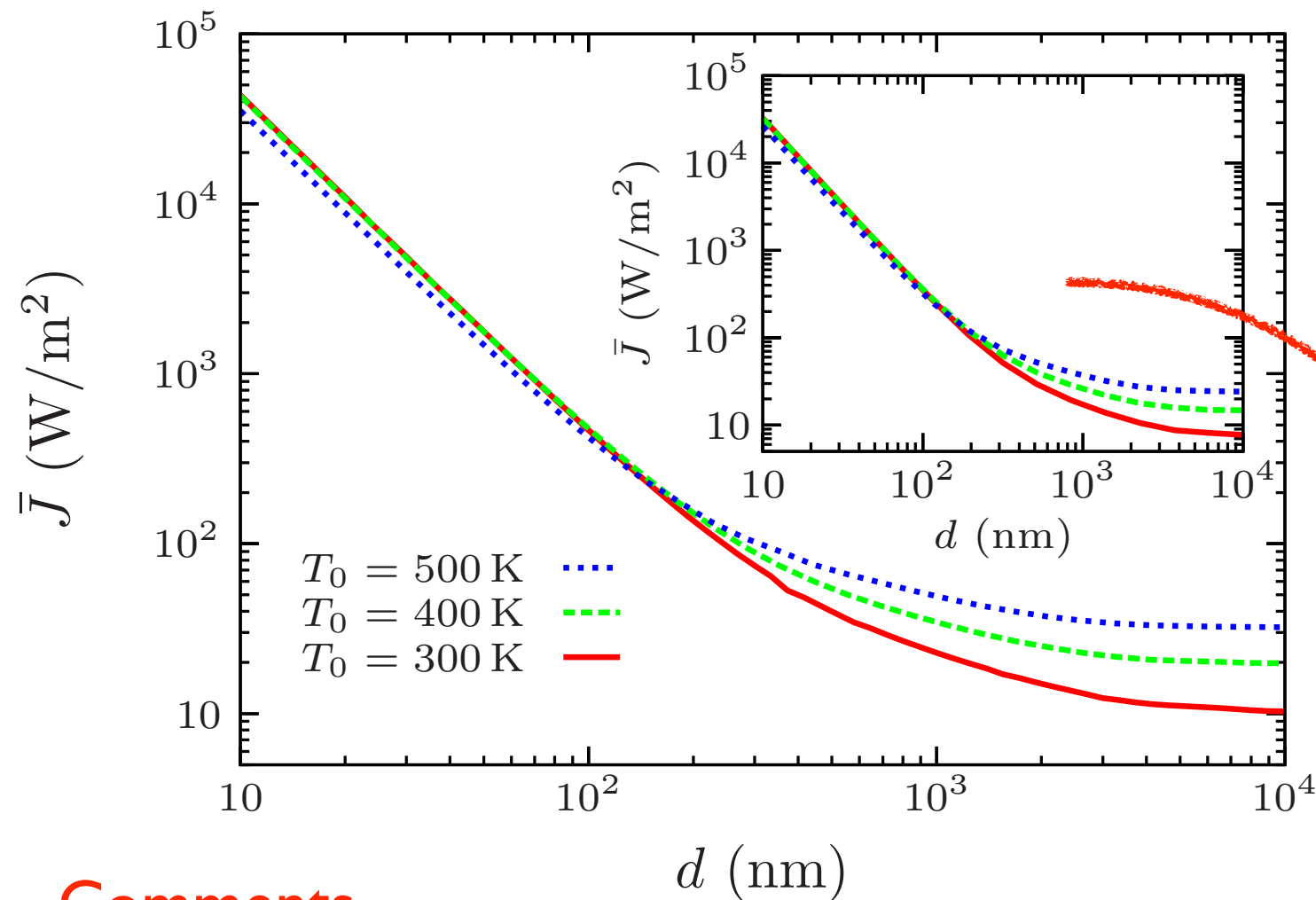
Case 2: optical properties depend on $X_L(t)$

The shuttling flux **can be negative**

Numerical results: two SiO₂ slabs

$$T_L(t) = T_0 + \delta T \sin(\Omega t)$$

$$\delta T = 30 \text{ K}$$



Main part of the plot

$$T_R(t) = T_0$$

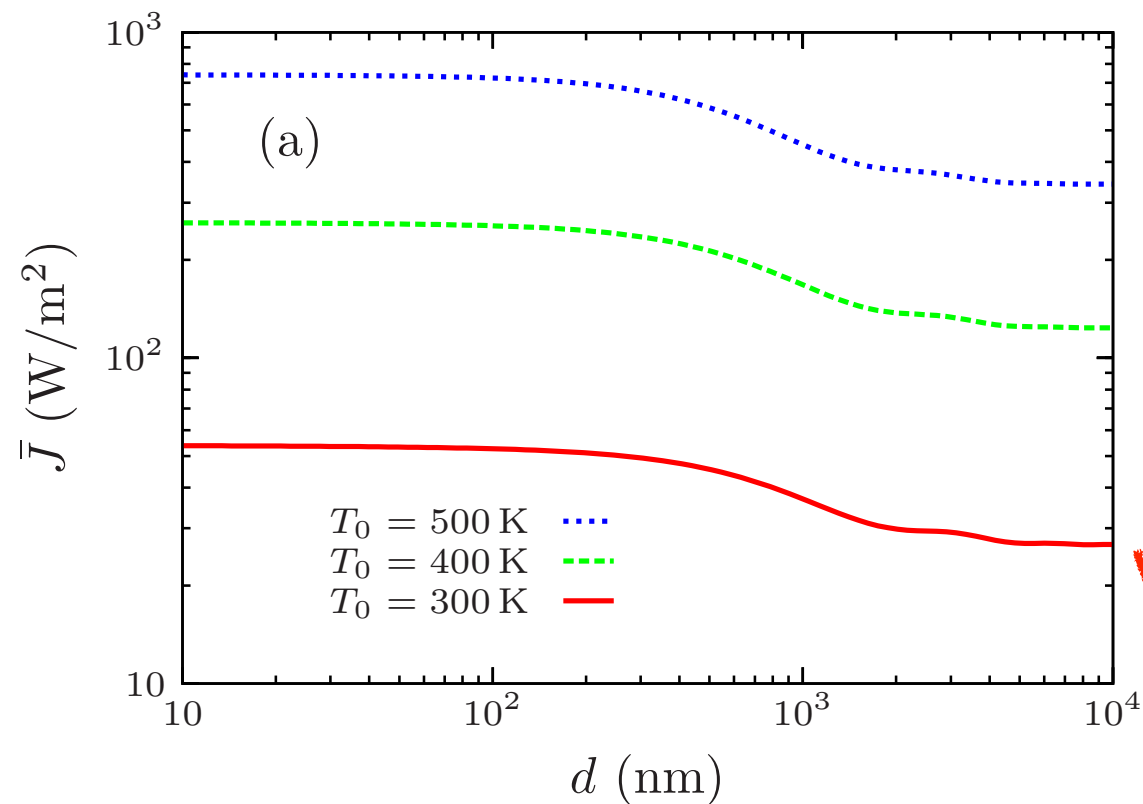
Inset

$$T_R(t) = T_0 + \frac{\delta T}{2} \sin(\Omega t)$$

Comments

- The shuttling flux is calculated **exactly**
- It is **always positive** as expected
- It **varies strongly with the distance** and goes as d^{-2} in the near field

Numerical results: InSb and SiO₂ slabs

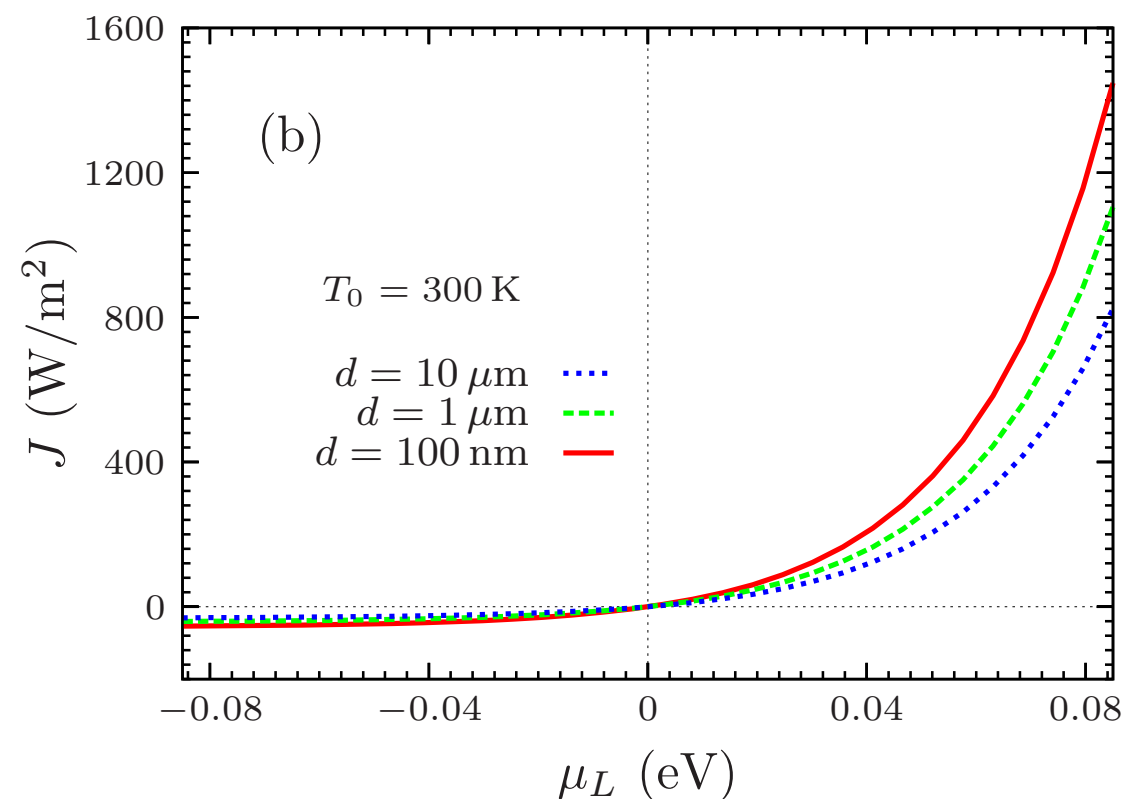


- Same temperature T_0
- InSb subject to an **oscillating voltage**

$$V_L(t) = \delta V \sin(\Omega t)$$

- Oscillating chemical potential

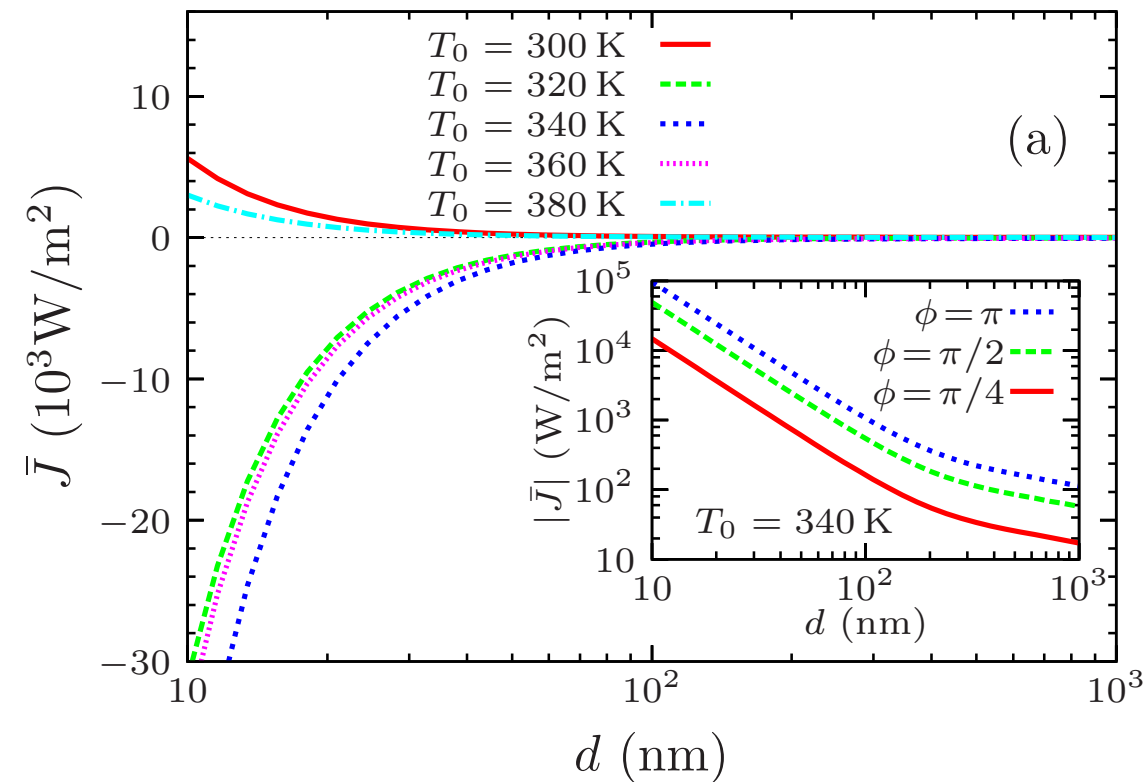
$$\mu_L(t) = q\delta V \sin(\Omega t)$$



Comments

- The average flux is **always positive**
- Strongly asymmetric **diode-like behavior** as a function of the chemical potential

Numerical results: VO₂ and SiO₂ slabs



- VO₂: **transition** around $T_c = 340 \text{ K}$ from a dielectric to a metallic behavior
- Varying temperature

$$T_L(t) = T_0 + \delta T \sin(\Omega t) \quad \delta T = 30 \text{ K}$$

Main part of the plot

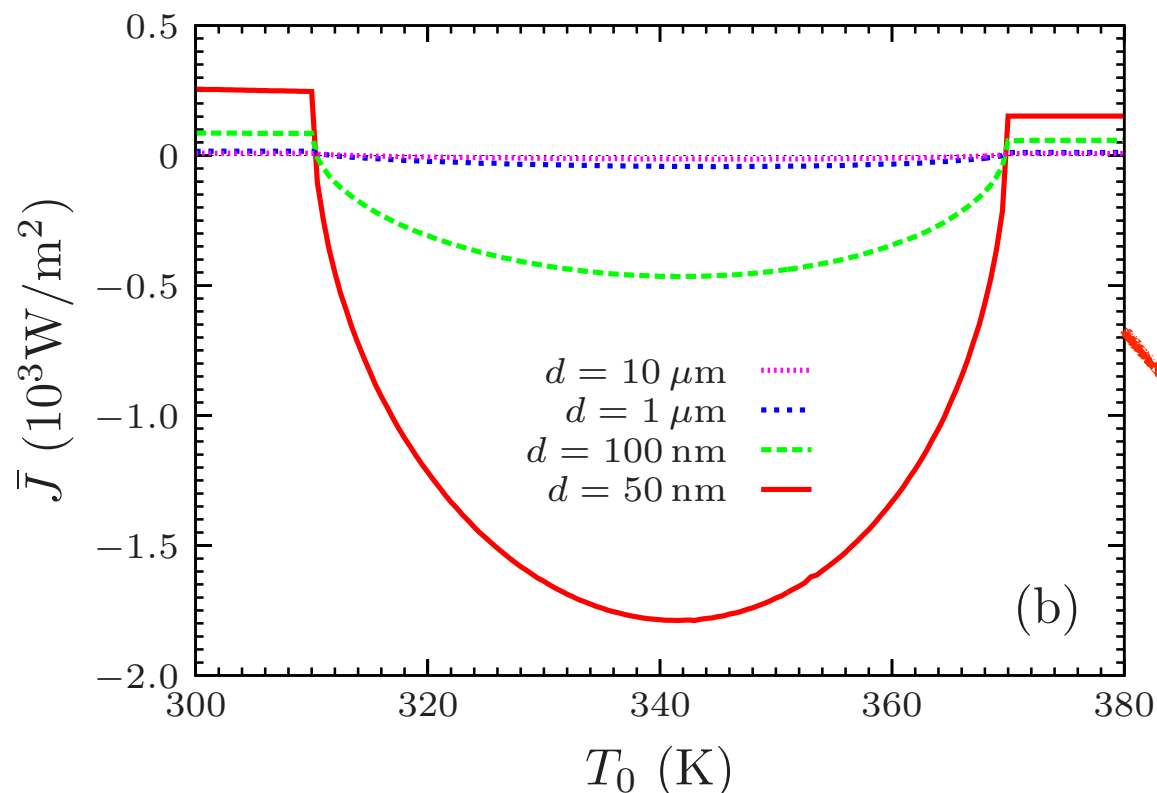
$$T_R(t) = T_0$$

The flux strongly depends on the distance and its **sign can be modulated** by changing the average temperature

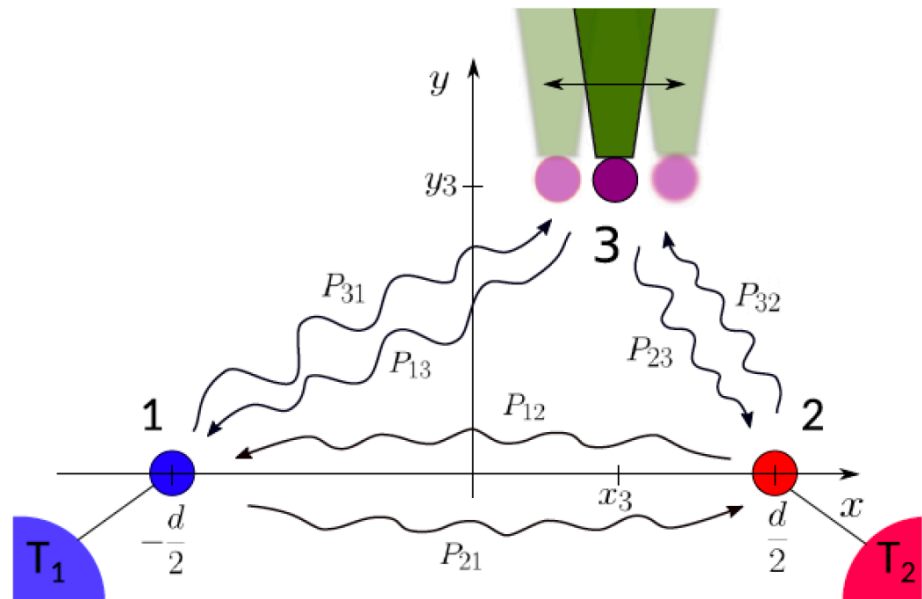
Inset

$$T_R(t) = T_0 + \delta T \sin(\Omega t + \phi)$$

$T_0 = T_c$ maximizes the shuttling flux



Radiative heat pumping



Third body acting as an **external probe**

Position and **temperature** varying as

$$T_3(t) = T_{3,\text{eq}} + \Delta T \sin(\omega t)$$

$$x_3(t) = \Delta x \sin(\omega t + \phi)$$

Treatment within the **dipolar approximation**

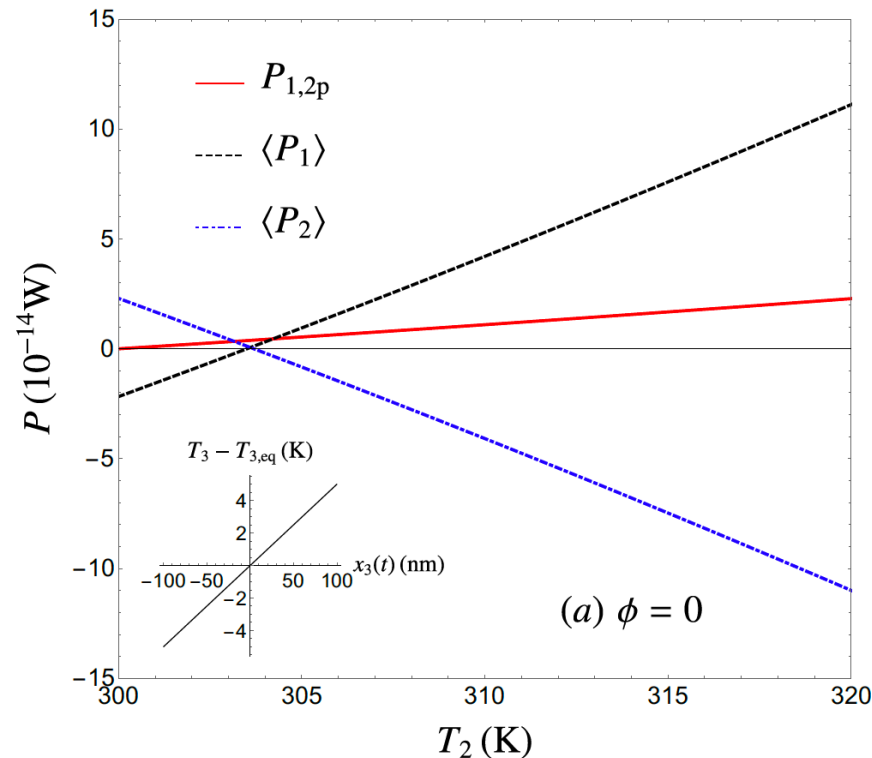
Calculation of the power absorbed by particles 1 and 2

Up to second order in ΔT and Δx

$$\langle P_1 \rangle \simeq \frac{\Delta T}{2} \left(\Delta x \frac{\partial^2 P_1}{\partial x_3 \partial T_3} \cos \phi + \frac{\Delta T}{2} \frac{\partial^2 P_1}{\partial T_3^2} \right)$$

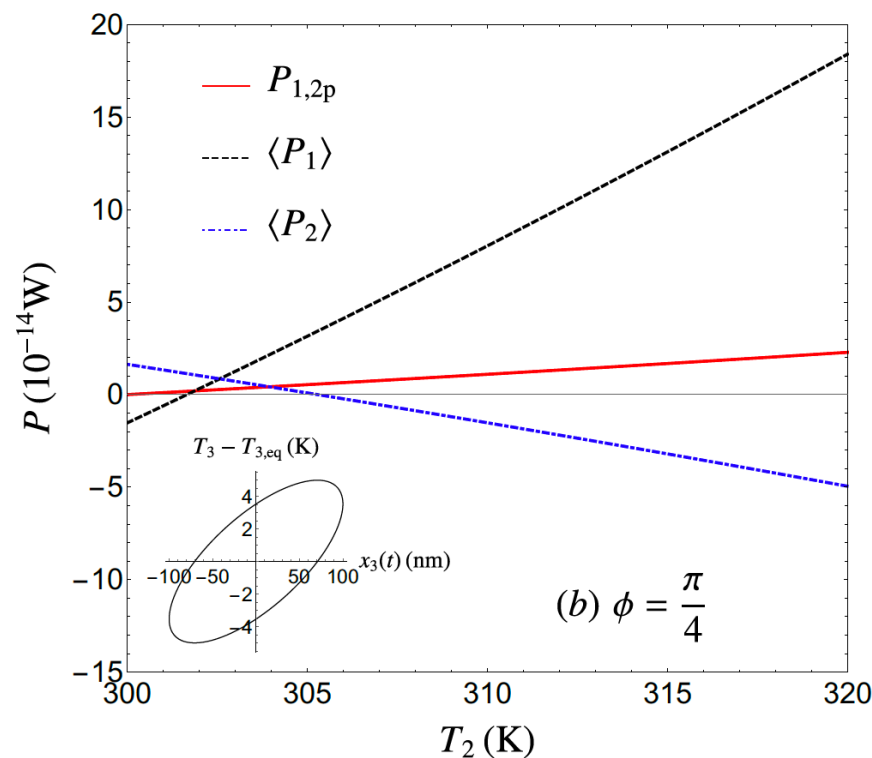
Dephasing acts as a control parameter on the value and sign of the absorbed power

Manipulating heat flux



Dephasing $\phi = 0$

- Two-body case
- **Heat-pumping effect** for $T_1 = T_2 = 300 \text{ K}$
- Induced equilibrium temperature $T > 300 \text{ K}$



Dephasing $\phi = \frac{\pi}{4}$

- Same two-body case
- Modified **heat-pumping effect**
- Different equilibrium temperatures

I. Latella et al., PRL **121**, 023903 (2018)

RM and P. Ben-Abdallah, PRB **101**, 165435 (2020)