Nanoscale Heat Radiation III

Boltzmann Equation, Topology, Non-Reciprocity

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Theorie der kondensierten Materie
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Heat transfer in many-body systems
Theoretical framework

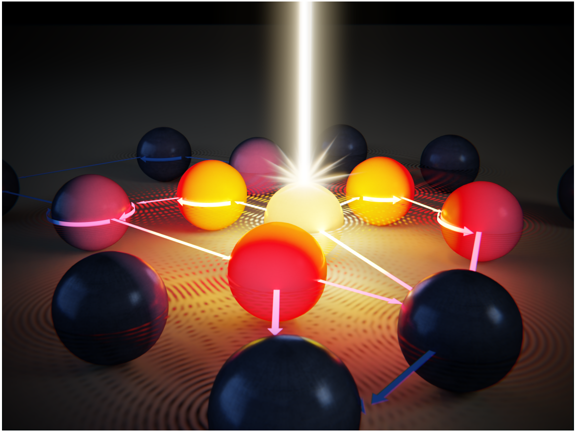
Heat transfer in a chain of nanoparticles
Boltzmann equation approach
Boltzmann vs Landauer

Topological many-body systems
Su-Schrieffer-Heeger model
Honeycomb lattice

Non-reciprocal many-body systems
Hall effect for thermal radiation

Summary

Thermal near-field radiation in many-body systems



Ben-Abdallah, SAB, Joulain, PRL **107**, 107, 114301 (2011)

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SAB, Messina, Venkataram, Rodriguez, Cuevas, Ben-Abdallah, Rev. Mod. Phys. **93**, 025009 (2021)

Thermal near-field radiation in many-body systems

- Power exchanged between particles with polarizability $\underline{\underline{\alpha}}$

$$\langle P_i \rangle = \sum_{j \neq i}^{N} 3 \int_0^\infty \frac{d\omega}{2\pi} \Big(\big[\Theta(\omega, T_j) - \Theta(\omega, T_i) \big] \mathcal{T}_{ji}(\omega) \Big)$$

- Mean energy of harmonic oscillator

$$\Theta(\omega, T) = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_\mathrm{B} T} - 1} \qquad \langle P_1 \rangle = 3 \left(\frac{\pi^2 k_\mathrm{B}^2 T}{3h} \right) \overline{\mathcal{T}}_{12} \Delta T_{12}$$

- Transmission coefficient (proportional to absorptivity $\frac{\underline{\underline{\alpha}} - \underline{\underline{\alpha}}^\dagger}{2i}$)

$$\mathcal{T}_{ij}(\omega) = \frac{4}{3} \mathrm{ImTr} \left[\boldsymbol{T}_{ij}^{-1} \frac{\underline{\underline{\alpha}} - \underline{\underline{\alpha}}^\dagger}{2i} (\boldsymbol{T}_{ij}^{-1})^\dagger \underline{\underline{\alpha}}^{-1\dagger} \right]$$

- 'Transfer matrix'

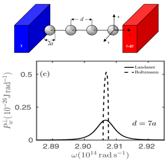
$$\boldsymbol{T}_{ij} = \delta_{ij} \mathbb{1} - (1 - \delta_{ij}) k_0^2 \underline{\underline{\alpha}} \mathbb{G}_{ij}^E$$

- Access to all observables: mean Poynting vector, energy density, etc.

SAB, Messina, Venkataram, Rodriguez, Cuevas, Ben-Abdallah, Rev. Mod. Phys. **93**, 025009 (2021)

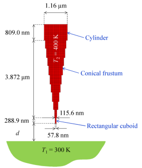
Application of many-body theory

Boltzmann vs. Landauer



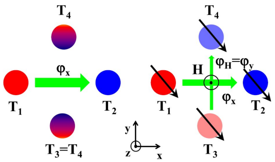
Ben-Abdallah, et al., PRL **111**, 174301 (2013)
Kathmann et al., PRB **98**, 115434 (2018)

Discrete Dipole Approximation



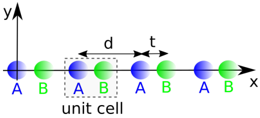
Edalatpour et al., JQSRT **133**, 364 (2014)
Edalatpour, Francoeur, PRB **94**, 045406 (2016)

non-reciprocal materials



Ben-Abdallah, PRL **116**, 084301 (2016)

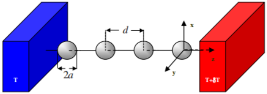
topological systems



Ott, SAB, PRB **102**, 115417 (2020)

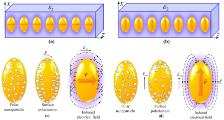
Boltzmann equation approach for plasmonics

chain of nanoparticles



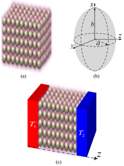
Ben-Abdallah, PRB **77**, 075417 (2008)

spheroidal nanoparticles



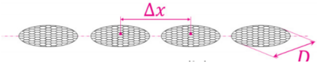
Ordonez-Miranda et al., PRB **92**, 115409 (2015)

plasmonic crystal



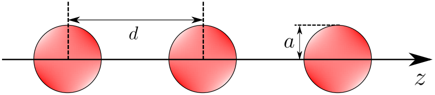
Ordonez-Miranda et al., PRB **93**, 035428 (2016)

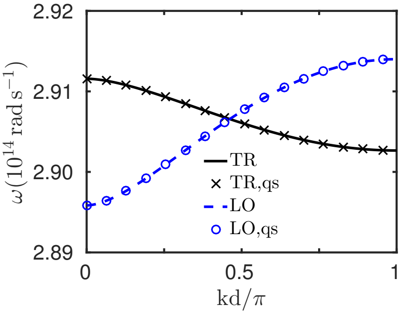
graphene disks



Ramirez et al., PRB **96**, 165428 (2017)

hBN nanoparticle chain (a = 25nm, d = 100nm)





Kathmann, Messina, Ben-Abdallah, SAB, PRB **98**, 115434 (2018)

- Induced dipole moment

$$\mathbf{p}_i = \epsilon_0 \alpha \mathbf{E}(\mathbf{r}_i)$$

- Field of other dipoles

$$\mathbf{E}(\mathbf{r}_i) = \mu_0 \omega^2 \sum_{j \neq i} \mathbb{G}(\mathbf{r}_i, \mathbf{r}_j)\mathbf{p}_j$$

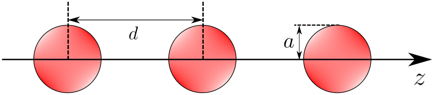
- Closed equation $\mu_0 \epsilon_0 = 1/c^2$

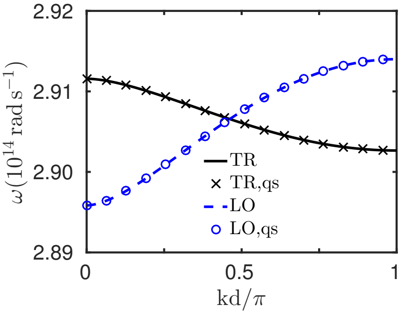
$$\frac{\omega^2}{c^2} \sum_{j \neq i} \mathbb{G}(\mathbf{r}_i, \mathbf{r}_j)\mathbf{p}_j = \frac{1}{\alpha}\mathbf{p}_i$$

- Bloch ansatz $\mathbf{p}_j = \mathbf{p}\mathrm{e}^{jkd}$
- Eigenvalue equation

$$\frac{\omega^2}{c^2} \sum_{j \neq i} \mathrm{e}^{(j-i)kd} \mathbb{G}(\mathbf{r}_i, \mathbf{r}_j)\mathbf{p} = \frac{1}{\alpha}\mathbf{p}$$

Boltzmann equation approach





Kathmann, Messina, Ben-Abdallah, SAB, PRB **98**, 115434 (2018)

- Heat current density

$$j = \frac{1}{LS}\sum_k (f_k - f_k^0) v_{g,k} \hbar\omega_k$$

- Boltzmann equation

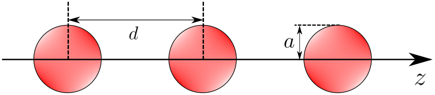
$$\frac{\partial f_k}{\partial t} + v_{g,k}\frac{\partial f_k}{\partial z} = -\frac{f_k - f_k^0}{\tau_k}$$

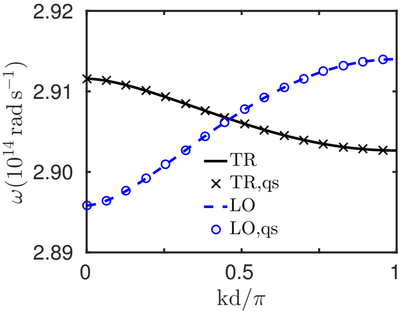
- Steady state

$$j \approx -\frac{1}{LS}\sum_k v_{g,k}^2 \tau_k \hbar\omega_k \frac{\partial f_k^0}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}$$

- Fourier's law $j = -\kappa\frac{\mathrm{d}T}{\mathrm{d}z}$

Boltzmann equation approach





Kathmann, Messina, Ben-Abdallah, SAB, PRB **98**, 115434 (2018)

- Replacing sum by integral ($\Lambda = v_g \tau$)

$$\sum_k v_k = \frac{L}{2\pi} \int \mathrm{d}k \frac{\mathrm{d}\omega_k}{\mathrm{d}k} = L \int \frac{\mathrm{d}\omega}{2\pi}$$

$$j \approx -\frac{1}{LS} \sum_k v_k^2 \tau_k \hbar\omega_k \frac{\partial f_k^0}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z}$$

$$= -\frac{2}{S} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \Lambda_\omega \frac{\partial \Theta(\omega)}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z}$$

- Transferred power ($\frac{\mathrm{d}T}{\mathrm{d}z} \approx \frac{\Delta T}{L}$)

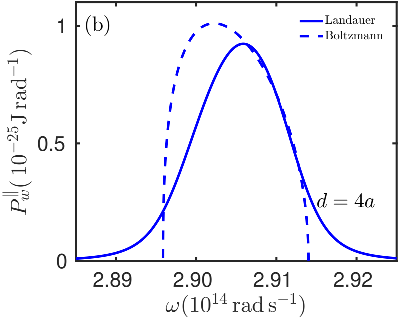
$$P_d = S|j| = \int \frac{\mathrm{d}\omega}{2\pi} \Theta_{1N} \frac{2}{L} (2\Lambda_\omega^\perp + \Lambda_\omega^\parallel)$$

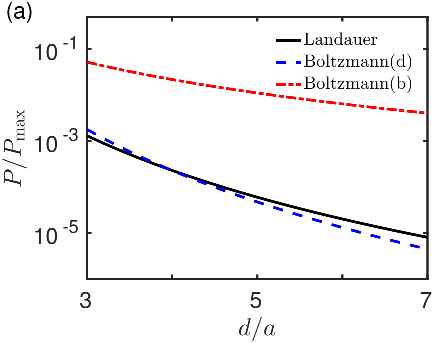
- Propagation length [$\exp(-\mathrm{i}\omega t)$]

$$\Lambda_\omega = \left| \frac{\mathrm{d}\omega'}{\mathrm{d}k} \frac{1}{2\omega''} \right|$$

Boltzmann vs. Landauer (hBN, a = 25nm)

- Spectral heat flux
- Full heat flux





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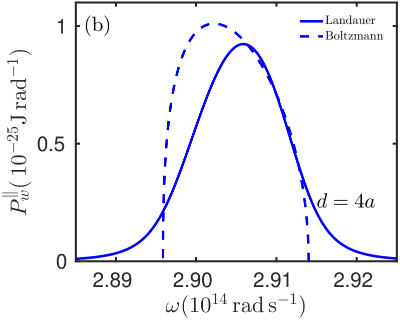
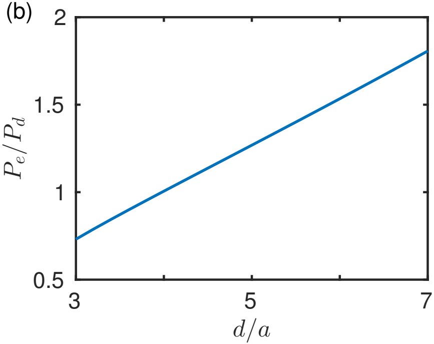


- Full heat flux



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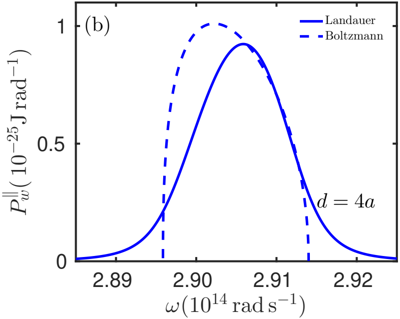
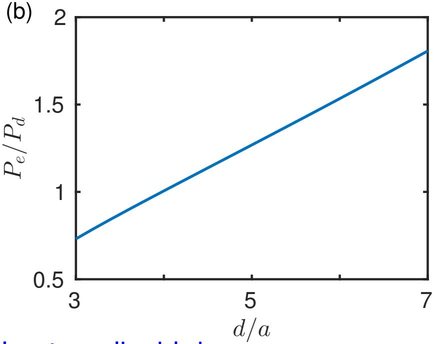


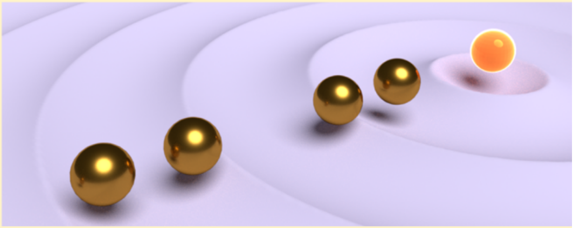


Boltzmann approach is in general not applicable!

see also Tervo et al, JQSRT 246, 106947 (2020)

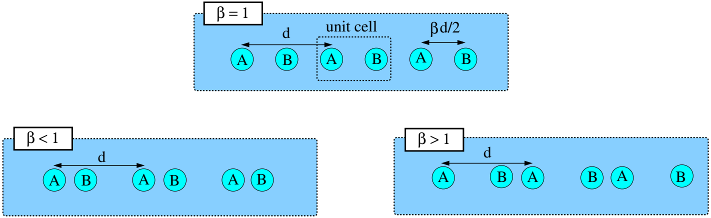
Kathmann, Messina, Ben-Abdallah, SAB, PRB **98**, 115434 (2018)

Su-Schrieffer-Heeger model in plasmonics



Pocock et al., ACS Photonics **5**, 2271 (2018)

Su-Schrieffer-Heeger model



- Bloch ansatz

$$\mathbf{M}\begin{pmatrix}\mathbf{p}_A\\ \mathbf{p}_B\end{pmatrix} = \frac{1}{\alpha}\begin{pmatrix}\mathbf{p}_A\\ \mathbf{p}_B\end{pmatrix}$$

- Form of the matrix **M**

$$\mathbf{M} = \begin{pmatrix} 0 & \mathbb{M}_{AB} \\ \mathbb{M}_{BA} & 0 \end{pmatrix} + \begin{pmatrix} \mathbb{M}_{AA} & 0 \\ 0 & \mathbb{M}_{AA} \end{pmatrix}$$

- Define "Hamiltonian" $\rightarrow$ SSH model

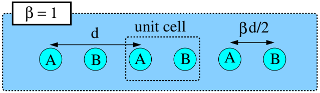
$$\mathbf{H}\begin{pmatrix}\mathbf{p}_A\\ \mathbf{p}_B\end{pmatrix} = \left(\frac{1}{\alpha} - M_{AA}\right)\begin{pmatrix}\mathbf{p}_A\\ \mathbf{p}_B\end{pmatrix}$$

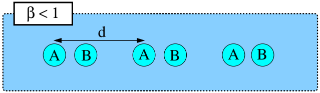
- Sublattice/Chiral symmetry

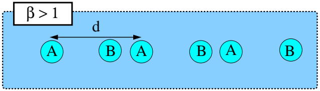
$$\sigma_z \mathbf{H} \sigma_z = -\mathbf{H}$$

Pocock et al., ACS Photonics **5**, 2271 (2018)

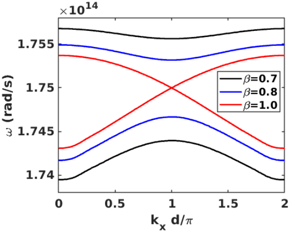
Nanoparticle chains (InSb, R = 100nm, d = 1μm)



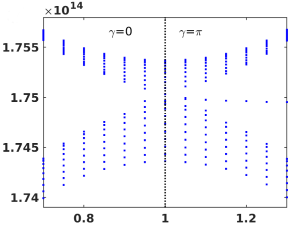




longitudinal modes ($N = \infty$)

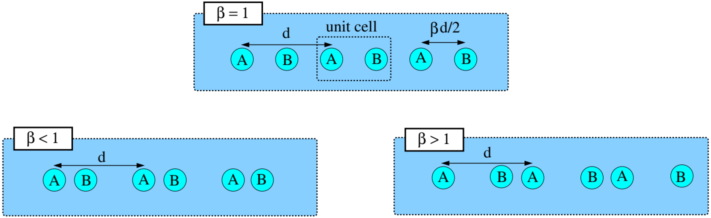


longitudinal modes ($N = 20$)



Ott, SAB, PRB **102**, 115417 (2020)

Winding number



- "Hamiltonian"

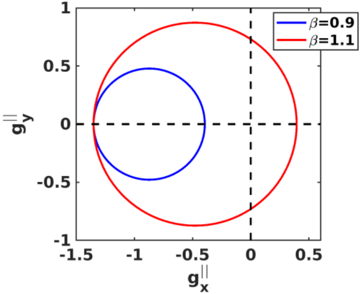
$$\mathbf{H} = \begin{pmatrix} 0 & M_{AB} \\ M_{BA} & 0 \end{pmatrix}$$

- Sublattice symmetry $\sigma_z \mathbf{H} \sigma_z = -\mathbf{H}$

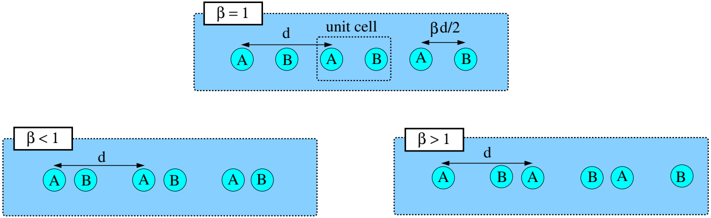
$$\mathbf{H} = \vec{\sigma} \cdot \vec{g}, \quad \vec{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}; \vec{g} = \begin{pmatrix} g_x \\ g_y \\ 0 \end{pmatrix}$$

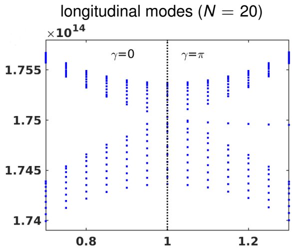
Ott, SAB, PRB **102**, 115417 (2020)

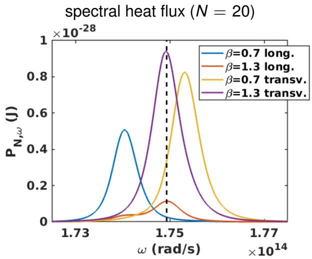
longitudinal modes ($N = \infty$)



Heat flux (R = 100nm, d = 1μm)

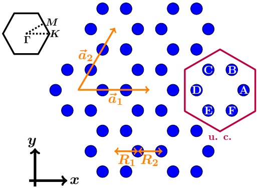


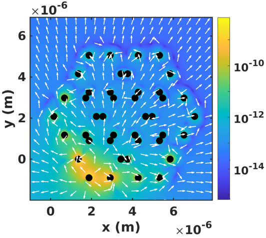


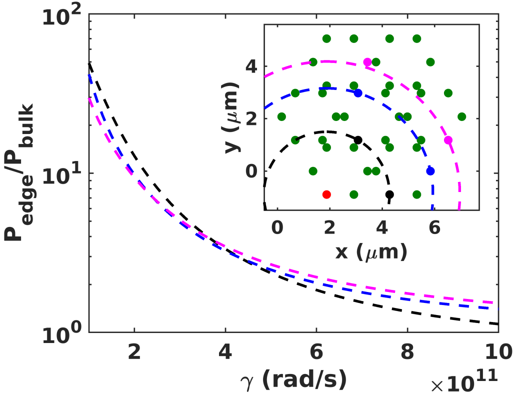


Ott, SAB, PRB **102**, 115417 (2020)

Edge mode heat transfer (R = 100nm, d = 1μm)

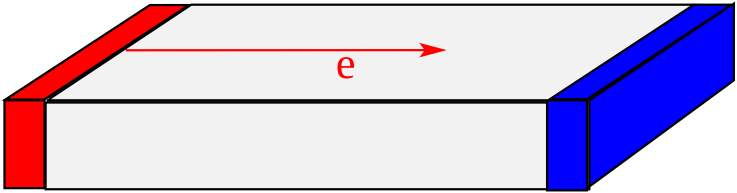






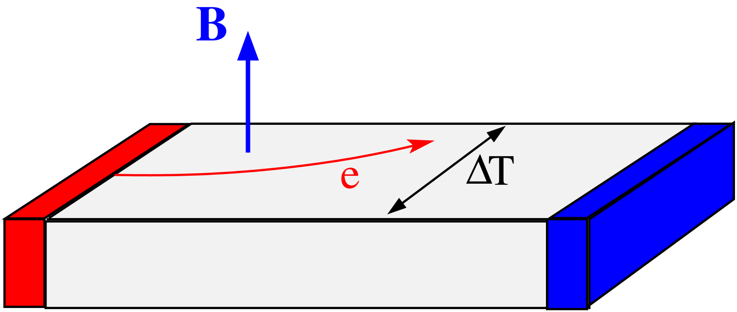
Ott, SAB, IJHMT **190**, 122796 (2022)

Righi-Leduc or thermal Hall effect



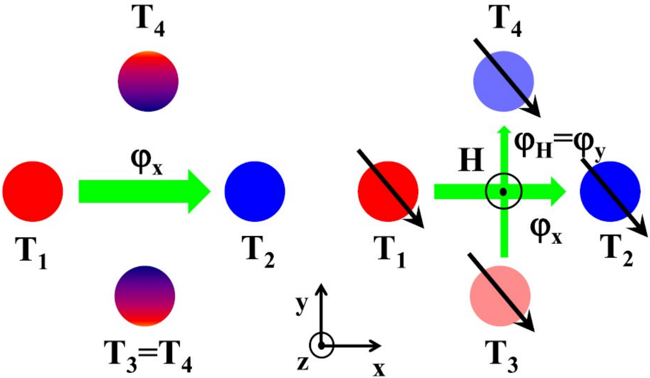
A. Righi, Mem. Acc. Lincei 4, 433 (1887); M. A. Leduc, J. Phys. 2e série 6, 378 (1887)

Righi-Leduc or thermal Hall effect



A. Righi, Mem. Acc. Lincei 4, 433 (1887); M. A. Leduc, J. Phys. 2e série 6, 378 (1887)

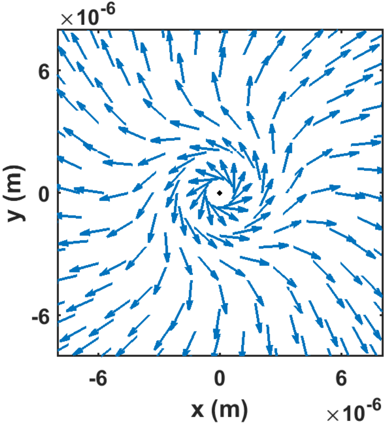
Hall effect for thermal radiation



Due to rotation of the optical axes?

Ben-Abdallah, PRL **116**, 084301 (2016)

Poynting vector around one InSb nanoparticle



- Polarizability

$$\underline{\underline{\alpha}} = 4\pi R^3 (\underline{\underline{\epsilon}} - \mathbb{1})(\underline{\underline{\epsilon}} + 2\mathbb{1})^{-1}$$

- Permittivity

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_1 & -i\epsilon_2 & 0 \\ i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

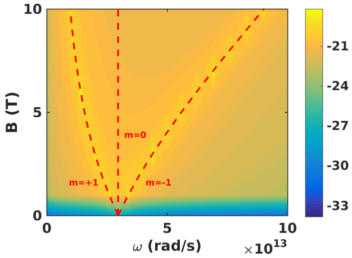
- Non-reciprocal

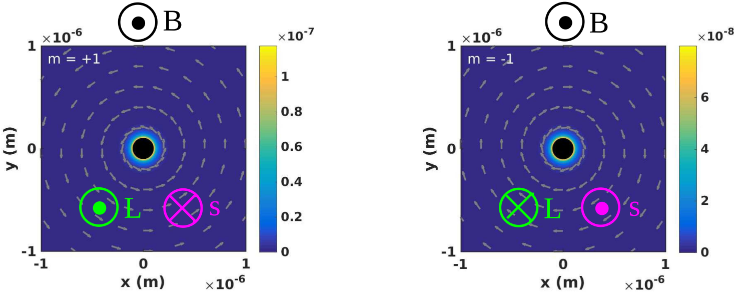
$$\underline{\underline{\alpha}}^t \neq \underline{\underline{\alpha}}, \quad \underline{\underline{\epsilon}}^t \neq \underline{\underline{\epsilon}}$$

$$S_\varphi(\omega) = \frac{\Theta(\omega, T_p) k_0^3}{2\pi^2 r^2} \mathrm{Re}(\alpha_{12}) \left(\frac{1}{k_0 r} + \frac{1}{k_0^3 r^3} \right)$$

Ott, Ben-Abdallah, SAB, PRB **97**, 205414 (2018)

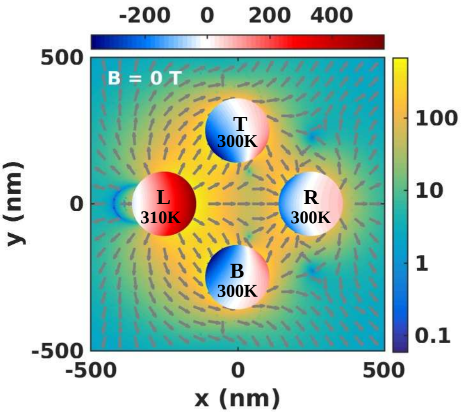
Dipolar resonances,($l = 1$, $m = -1, 0, +1$)





Ott, Ben-Abdallah, SAB, PRB **97**, 205414 (2018)

Heat flux and emmitted/received power ($R = 100\,\mathrm{nm}$, $d = 500\,\mathrm{nm}$)



Ott, Ben-Abdallah, SAB, PRB **97**, 205414 (2018)

Ott, Messina, Ben-Abdallah, SAB, J. Photon. Energy **9**, 032711 (2019)

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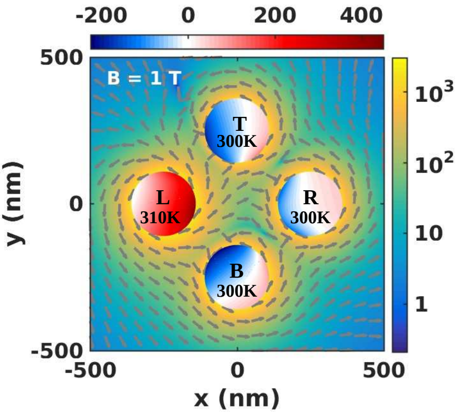

Ott, Ben-Abdallah, SAB, PRB **97**, 205414 (2018)

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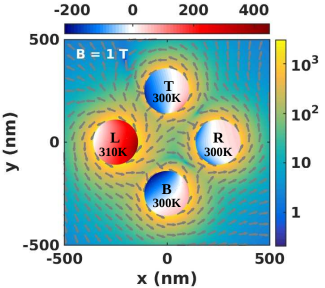
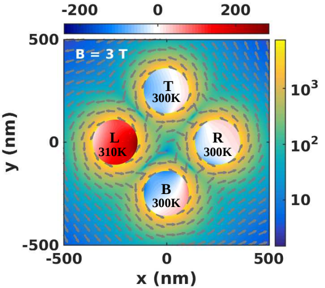
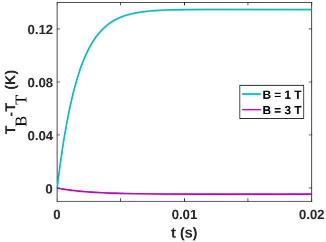
Photonic thermal Hall effect







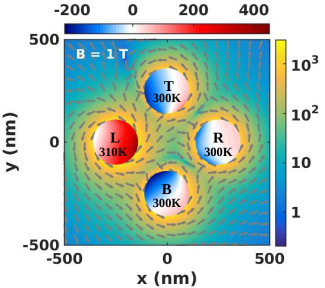
"Righi-Leduc coefficient"

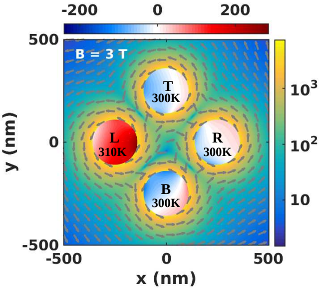
$$R_\mathrm{T} = \frac{T_\mathrm{B}^{(st)} - T_\mathrm{T}^{(st)}}{T_\mathrm{L} - T_\mathrm{R}}$$

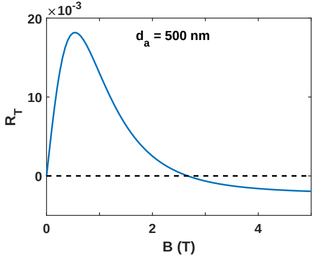
Ott, Messina, Ben-Abdallah, SAB, J. Photon. Energy **9**, 032711 (2019)

Anomalous Hall effect for Weyl semi metals: Ott, SAB, Ben-Abdallah, PRB **101**, 241411(R) (2020)

Photonic thermal Hall effect







"Righi-Leduc coefficient"

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Ott, Messina, Ben-Abdallah, SAB, J. Photon. Energy **9**, 032711 (2019)

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Summary

- Near-field heat radiation in many-body systems
  - Chains of nanoparticles
    - Boltzmann vs. Landauer
    - Boltzmann can fail for dielectrics
    - Boltzmann certainly fails for metals
  - Topological systems
    - Su-Schrieffer-Heeger (SSH) chain
    - Honeycomb lattice
    - Edge mode dominated heat transfer
- Nonreciprocal systems
  - Circular heat flux, angular momentum, spin
  - Hall effect for thermal radiation

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