Nanoscale Heat Radiation I

Planck law and Stefan-Boltzmann law reloaded

Svend-Age Biehs

Theorie der kondensierten Materie
Institut für Physik
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Frejus, May 11 2022

rerum cognoscere causas

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Introduction

- History
- Generalized Planck law
- Scanning noise microscope (SNoiM)

Fluctuational electrodynamics

- Theoretical framework
- Near-field heat flux
- Landauer-form

Applications

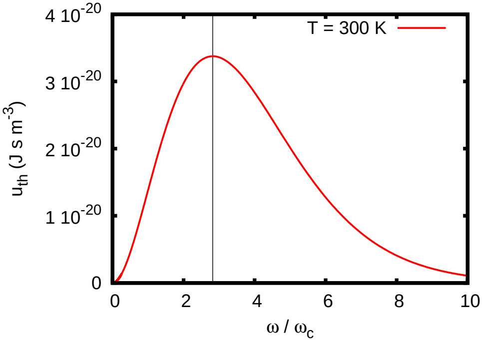
- Overview

Summary

Blackbody radiation

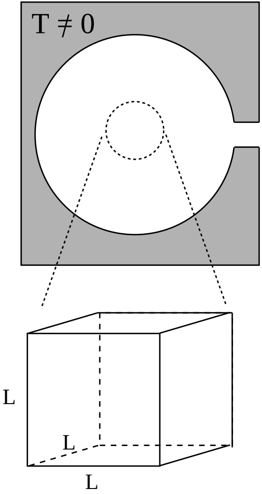
Max Planck, 1901

(DOI: 10.1115/HT2009-88060)



- Broadband spectrum
- Small temporal coherence ($\tau_c^{300K} = 25\,\mathrm{fs}$)
- Small spatial coherence ($l_c^{300K} = c\tau_c^{300K} = 6.7\,\mu\mathrm{m}$)

Planck law and Stefan-Boltzmann law



- Energy density in equilibrium

$$u^{\rm BB} = \frac{\epsilon_0}{2}\langle \hat{\mathbf{E}}\cdot\hat{\mathbf{E}}\rangle_{\rm th} + \frac{\mu_0}{2}\langle \hat{\mathbf{H}}\cdot\hat{\mathbf{H}}\rangle_{\rm th}$$

- Planck law

$$u_\omega^{\rm BB} = \frac{\hbar\omega}{{\rm e}^{\frac{\hbar\omega}{k_B T}} - 1}\frac{\omega^2}{\pi^2 c^3}$$

- Stefan-Boltzmann law

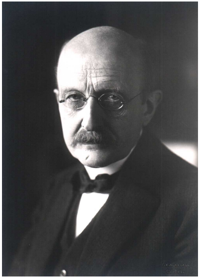
$$\Phi_{\rm BB} = \frac{c}{4}\int {\rm d}\omega\, u_{\rm BB}(\omega) = \sigma T^4$$

- Kirchhoff law:

emissivity = absorptivity

$\rightarrow$ No real emitter can emit more heat than a blackbody!

On the validity of Planck's theory

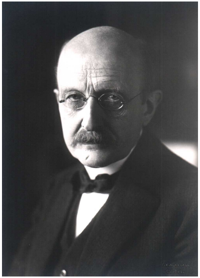


Max Planck, 1930

Throughout the following discussion it will be assumed that the linear dimensions of all parts of space considered, as well as the radii of curvature of all surfaces under consideration, are large compared with the wave lengths of the rays considered . . .

The theory of heat radiation, 1913

On the validity of Planck's theory



Max Planck, 1930

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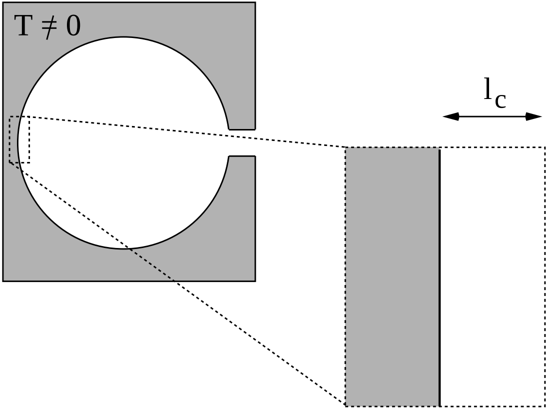
The theory of heat radiation, 1913

What if distances are smaller than $\lambda_{\rm th}$ or l_c?

Planck law in near-field regime?



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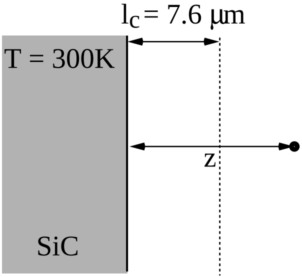
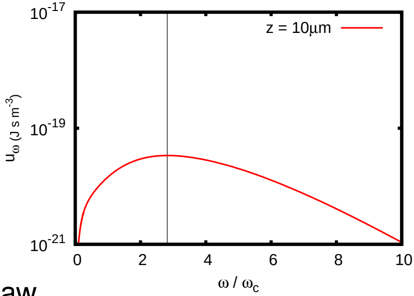
G.S.Agarwal, PRA **11**, 230 (1975), G.S.Agarwal, PRA **11**, 253 (1975)

Generalized Planck



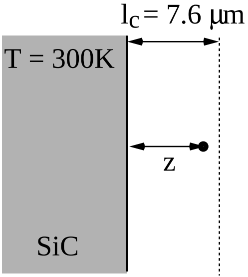


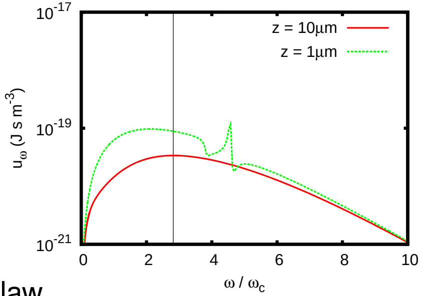
- Generalized Planck law

$$u_\omega = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1} D(\omega, z)$$

- Near-field properties
  -
  -
  -
  -

Generalized Planck





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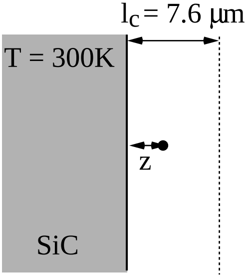

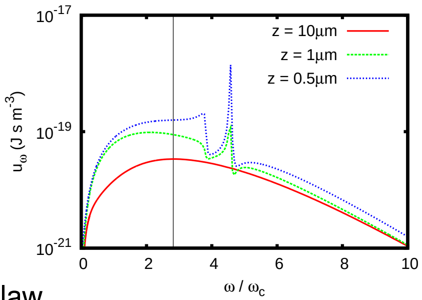

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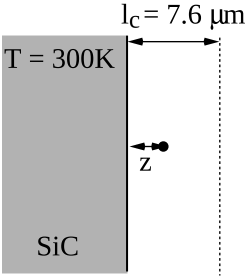

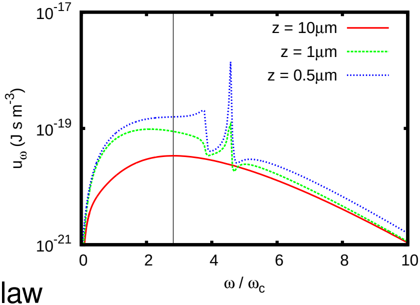

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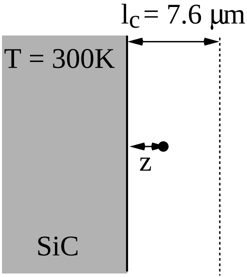
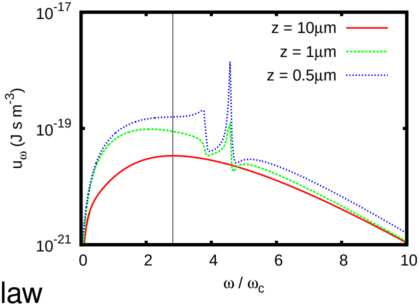
  - enhanced energy density (Eckhardt, Z. Phys. B **46**, 85 (1982))

Generalized Planck





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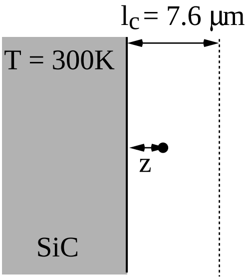
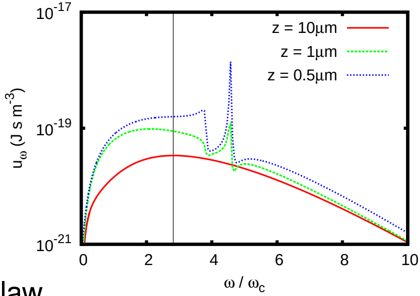
Generalized Planck





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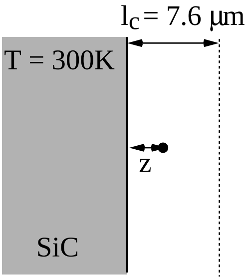
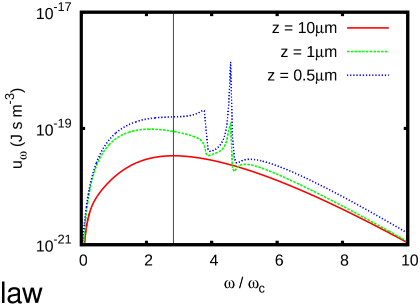
  - larger temporal coherence (Shchegrov et al., PRL **85**, 1548 (2000))
  -

Generalized Planck





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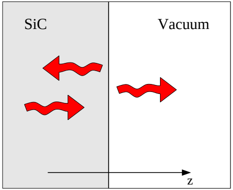
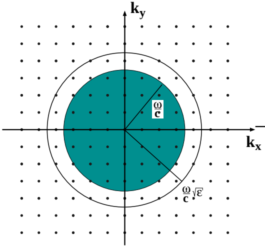
  - larger temporal coherence (Shchegrov et al., PRL **85**, 1548 (2000))
  - larger spatial coherence (Carminati, Greffet, PRL **82**, 1660 (1999))

Physics behind generalized Planck law

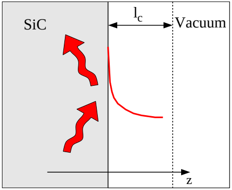
propagating waves



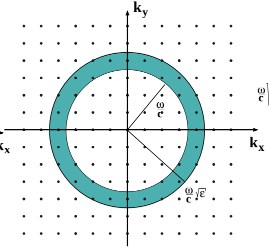
$$k_x^2 + k_y^2 \leq \frac{\omega^2}{c^2}$$



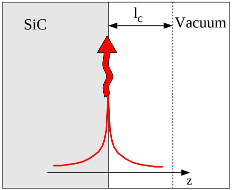
total int. reflection



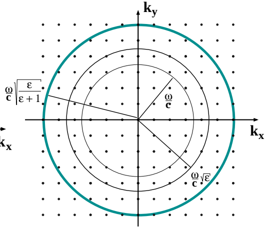
$$\frac{\omega^2}{c^2} < k_x^2 + k_y^2 \leq \frac{\omega^2}{c^2}\epsilon$$



surface waves

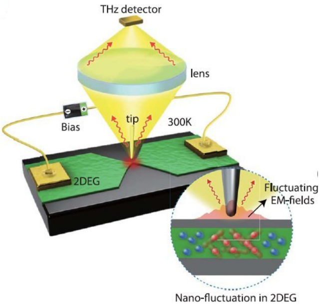


$$k_x^2 + k_y^2 = \frac{\omega^2}{c^2}\frac{\epsilon}{\epsilon+1}$$

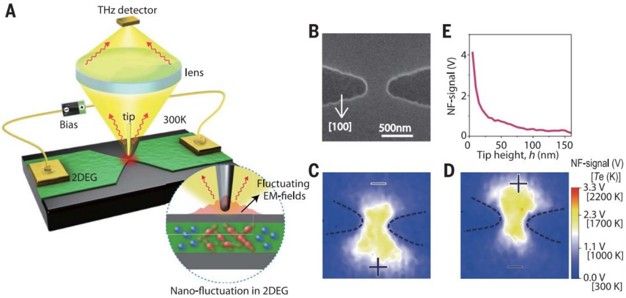


Scanning noise microscope (SNoiM)

Susumu Komiyama
(Tokyo University)



Measuring the near-field energy density



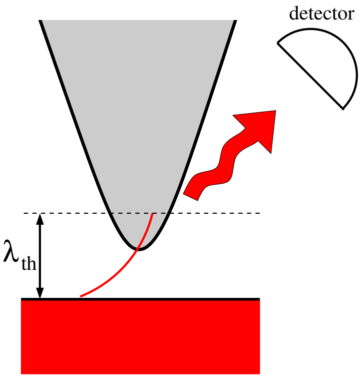
Weng, Komiyama, Yang, An, Chen, SAB, Kajihara, Lu, Science **360**, 775 (2018)

- Signal at the detector $\propto \langle|\mathbf{E}_{\rm sc}|^2\rangle \propto |\alpha|^2\langle|\mathbf{E}(\mathbf{r}_{\rm tip})|^2\rangle$
- Sensitive to the energy density

$$u_E = \frac{\hbar\omega}{e^{\hbar\omega/k_B T_{\rm sample}} - 1} D_E(\omega, \mathbf{r}_{\rm tip})$$

- Measurement of the LDOS and $T_{\rm sample}$

Theoretical model for SNoiM



Joulain et al., JQSRT **136**, 1 (2014)

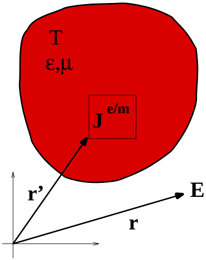
Herz, SAB, JQSRT **266**, 107572 (2021)

Herz, SAB, arXiv:2112.12016 (2022)

- Power at detector
  $$P = P^{\text{sub}} + P^{\text{eq}} + P_1^{\text{leq}} + P_2^{\text{leq}}$$
- Emission of substrate
  $$P^{\text{sub}} \propto (T_s - T_b)(1 - |r|^2)$$
- Emission of NP
  $$P^{\text{eq}} \propto (T_p - T_b)\mathrm{Im}(\alpha)$$
- Absorption in NP
  $$P_1^{\text{leq}} \propto (T_b - T_s)\mathrm{Im}(\alpha)$$
- Scattering by NP
  $$P_2^{\text{leq}} \propto (T_s - T_b)|\alpha|^2$$

How to treat out of equilibrium problems?

Rytov's fluctuational electrodynamics



- Maxwell's eqs. + fluct. source currents

$$\langle \mathbf{J} \rangle = \mathbf{0}$$

- Fluctuating fields

$$E_\alpha(\omega, \mathbf{r}) = \mathrm{i}\omega\mu_0 \int_V \mathrm{d}^3 r' \, \mathbb{G}^{\mathrm{EE}}_{\alpha\beta}(\mathbf{r}, \mathbf{r}') J_\beta(\mathbf{r}', \omega)$$

- Fluctuation-dissipation theorem

$$\langle J_\alpha(\mathbf{r}) J^*_\beta(\mathbf{r}') \rangle = \Theta(\omega, T) 2\omega\epsilon_0 \left[\mathrm{Im}(\epsilon_{\alpha\beta}) \delta(\mathbf{r} - \mathbf{r}') \right]$$

- Mean energy of a harmonic oscillator

$$\Theta(\omega, T) := \frac{\hbar\omega}{2} + \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_\mathrm{B} T} - 1}$$

Field correlation functions

- Correlation functions (applying the FDT)

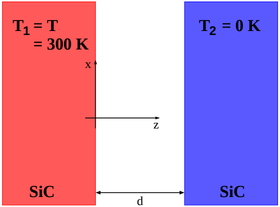
$$\langle E_\alpha(\mathbf{r},t)E_\beta(\mathbf{r}',t')\rangle = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\mathrm{e}^{\mathrm{i}\omega(t-t')}2\mu_0^2\omega^3\Theta(\omega,T)\int_V \mathrm{d}^3r'' \times \left(\epsilon_0\mathbb{G}^{\mathrm{EE}}(\mathbf{r},\mathbf{r}'')\underline{\underline{\epsilon}}''(\mathbf{r}'')\mathbb{G}^{\mathrm{EE}^\dagger}(\mathbf{r}',\mathbf{r}'')\right)_{\alpha\beta}$$

$$\langle E_\alpha(\mathbf{r},t)H_\beta(\mathbf{r}',t')\rangle = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\mathrm{e}^{\mathrm{i}\omega(t-t')}2\mu_0^2\omega^3\Theta(\omega,T)\int_V \mathrm{d}^3r'' \times \left(\epsilon_0\mathbb{G}^{\mathrm{EE}}(\mathbf{r},\mathbf{r}'')\underline{\underline{\epsilon}}''(\mathbf{r}'')\mathbb{G}^{\mathrm{HE}^\dagger}(\mathbf{r}',\mathbf{r}'')\right)_{\alpha\beta}$$

etc.

- Symmetrically ordered operators
- Stress tensor: Casimir-Van der Waals forces (Lifshitz, JETP **2**, 73 (1956))

Heat flux expression
Polder and van Hove, PRB **4**, 3303 (1971)

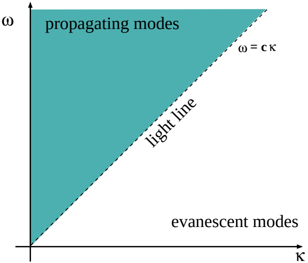


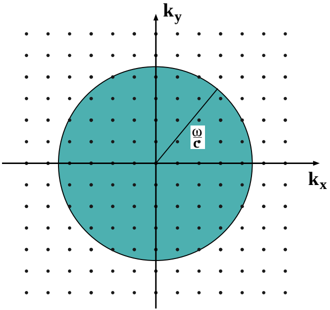
$$
\begin{aligned}
\Phi &= \langle S_z \rangle \\
&= \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/(k_\mathrm{B} T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \left(\mathcal{T}_\mathrm{s} + \mathcal{T}_\mathrm{p}\right)
\end{aligned}
$$

- Transmission coefficient

$$
\mathcal{T}_i(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \dfrac{(1-|r_i^{10}|^2)(1-|r_i^{20}|^2)}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa < \frac{\omega}{c} \\ \dfrac{4\mathrm{Im}(r_i^{10})\mathrm{Im}(r_i^{20}) e^{-2|k_z|d}}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa > \frac{\omega}{c} \end{cases}
$$

Propagating modes





- Plane wave

$$E_y = A(x, y; t)\mathrm{e}^{\mathrm{i}k_z z}$$

- Prop. modes $\kappa < \frac{\omega}{c}$

$$k_z = \sqrt{\frac{\omega^2}{c^2} - \kappa^2} \in \mathbb{R}$$

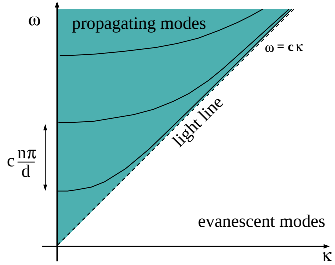
- Evan. modes $\kappa > \frac{\omega}{c}$

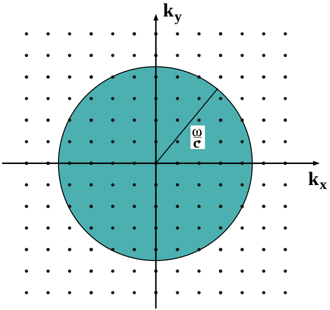
$$k_z = \mathrm{i}\sqrt{\kappa^2 - \frac{\omega^2}{c^2}} \in \mathbb{C}$$

- Res. transmission

$$k_z \equiv \frac{n\pi}{d}, n \in \mathbb{N}$$

Propagating modes





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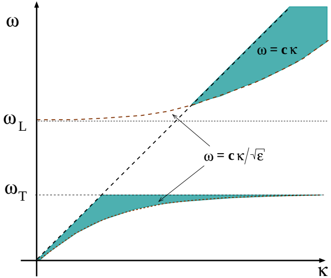
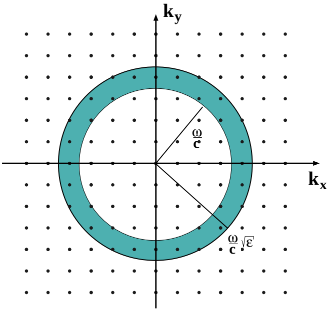
Frustrated internal reflection

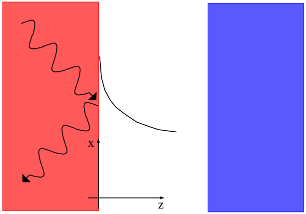




- Propagating waves inside the medium

$$k_{1,z} = \sqrt{\frac{\omega^2}{c^2}\epsilon - \kappa^2} \in \mathbb{R}$$

$$\Leftrightarrow \kappa < \frac{\omega}{c}\sqrt{\epsilon}$$



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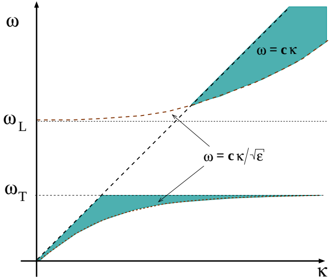

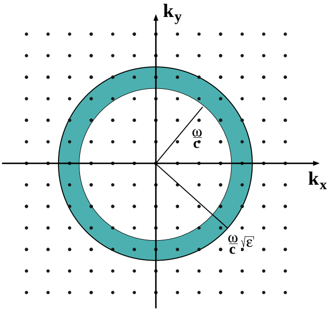

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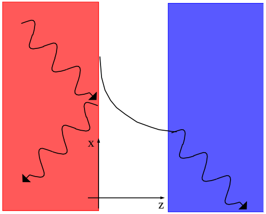
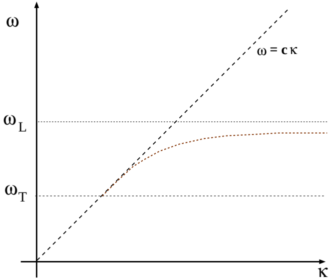
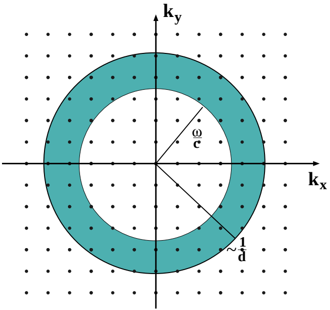


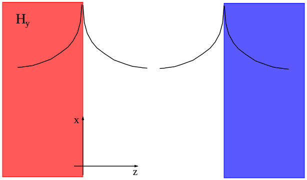
Surface modes



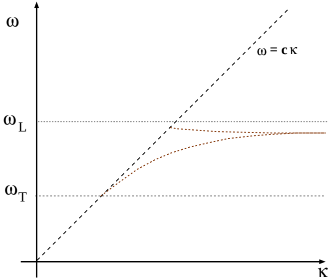


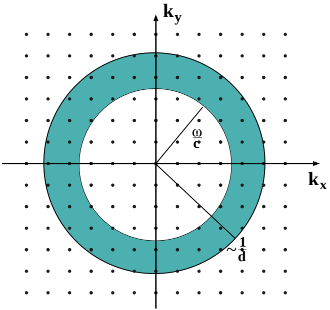
- 'bound' to the surface
- p-polarisation only
- Transmission coeff. $\kappa \gg \omega/c$

$$\mathcal{T}_{\rm p} \approx \frac{4\mathrm{Im}(r_{\rm p}^{10})\mathrm{Im}(r_{\rm p}^{20})e^{-2\kappa d}}{|1 - r_{\rm p}^{10} r_{\rm p}^{20} \exp(-2\kappa d)|^2}$$



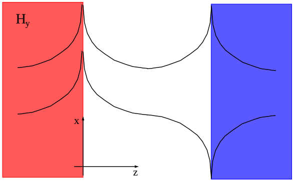
Surface modes



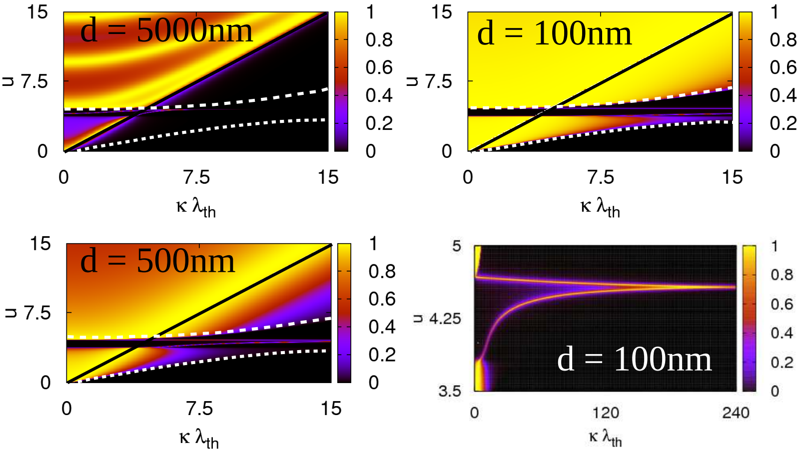


- 'bound' to the surface
- p-polarisation only
- Transmission coeff. $\kappa \gg \omega/c$

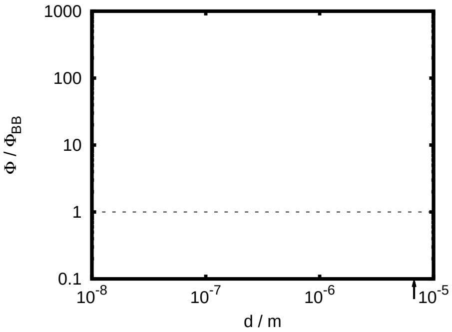
$$\mathcal{T}_\mathrm{p} \approx \frac{4\mathrm{Im}(r_\mathrm{p}^{10})\mathrm{Im}(r_\mathrm{p}^{20})e^{-2\kappa d}}{|1 - r_\mathrm{p}^{10} r_\mathrm{p}^{20} \exp(-2\kappa d)|^2}$$

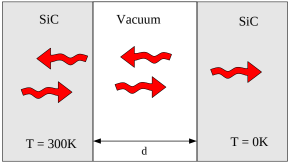


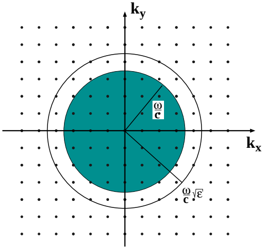
Transmission coefficient (SiC, $u = \hbar\omega / k_B T$)



Consequences for radiative heat flux







Φ_{BB} = blackbody value

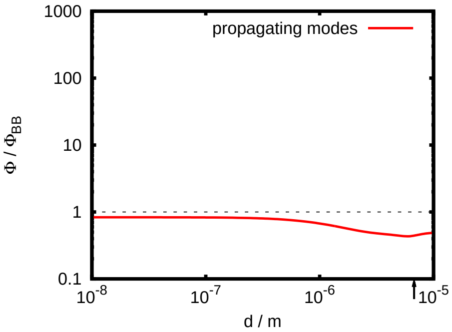
Polder and van Hove, PRB **4**, 3303 (1971)

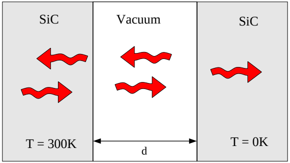
Pendry, J. Phys. **11** 6621 (1999)

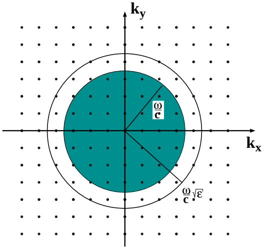
Volokitin and Persson, RMP **79**, 1291 (2007)

SAB, Rousseau, Greffet, PRL **105**, 234301 (2010)

Consequences for radiative heat flux







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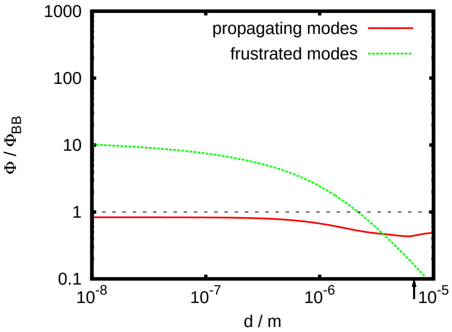

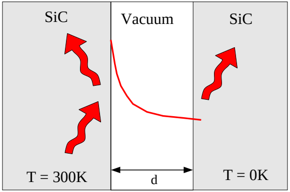

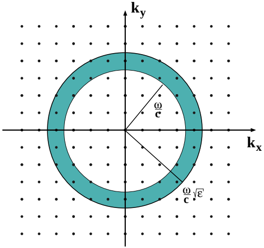

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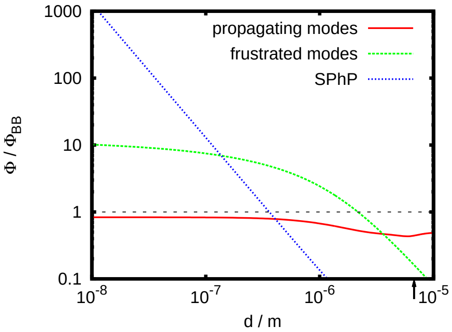

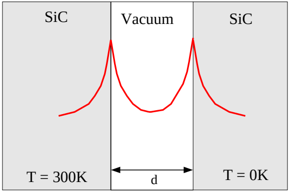

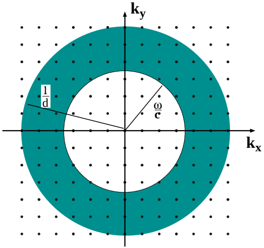

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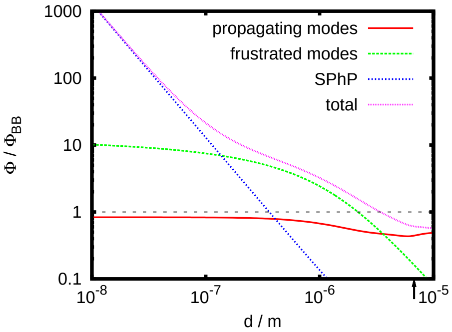
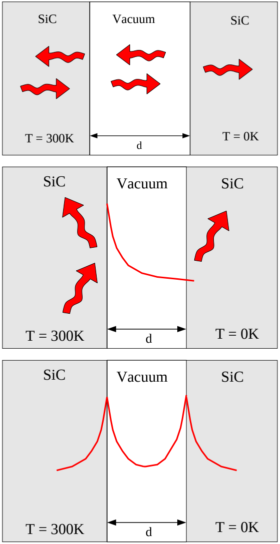
Consequences for radiative heat flux





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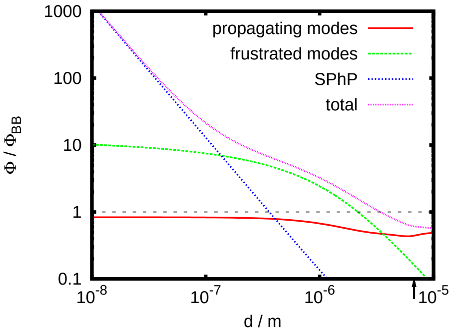

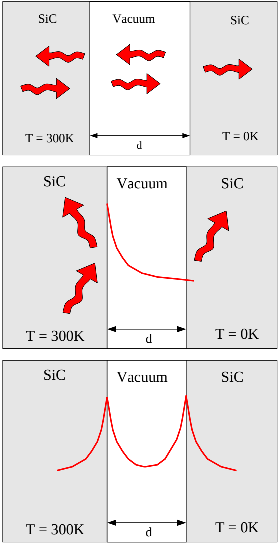

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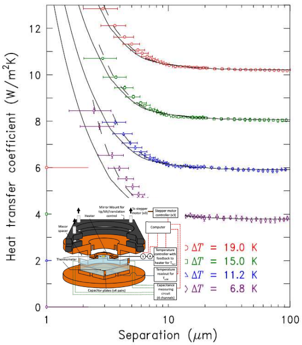
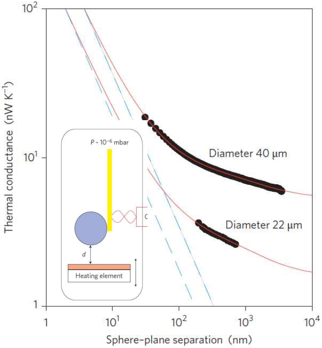
SAB, Rousseau, Greffet, PRL **105**, 234301 (2010)

Stefan-Boltzmann law is no limit in the near-field regime!

Experimental verification





- Hu et al., APL **92**, 133106 (2008)

  Ottens et al., PRL **107**, 014301 (2011)

  Kralik et al., PRL **109**, 224302 (2012)

  Song et al., Nat. Nanotech. **11**, 509 (2016)

  Bernardi et al., Nat. Comm. **7**, 12900 (2016)

  M. Ghashami, PRL **120**, 175901 (2018)

- Shen et al., Nano Lett. **9**, 2909 (2009)

  Rousseau et al., Nat. Photonics **3**, 514 (2009)

  van Zwol et al., PRL **108**, 234301 (2012)

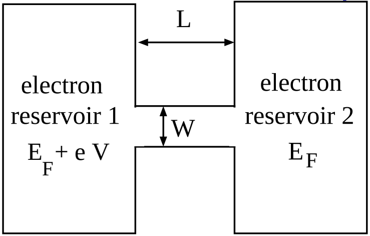
  Song et al., Nat. Nanotech. **10**, 253 (2015)

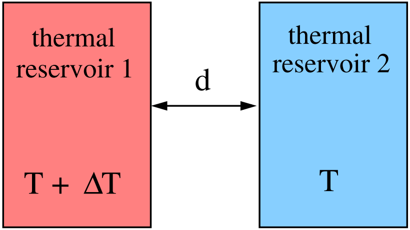
  Kim et al., Nature **528**, 387 (2015)

  Cui et al., Nat. Comm. **8**, 14479 (2017)

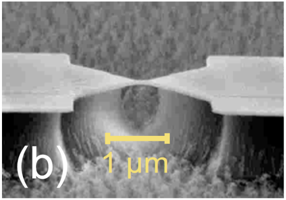
$700\times\Phi_{BB}$ at 25nm! Fiorino et al. Nano Lett. **18**, 3711 (2018)

Landauer-like expression for the heat flux





$$I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V \qquad \Phi = \frac{\pi^2 k_{\rm B}^2 T}{3h}\left[\sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_i\right] \Delta T$$



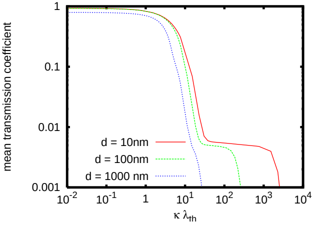
Wu et al., PRB **78**, 235421 (2008)

$$\overline{\mathcal{T}}_i = \frac{\int \mathrm{d}u\, u^2 f(u) \mathcal{T}_i(u,d)}{\int \mathrm{d}u\, u^2 f(u)}$$

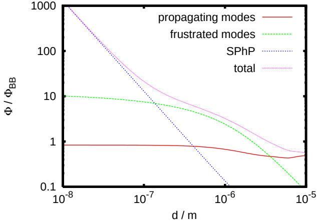
$$f(u) = \frac{u^2 \mathrm{e}^u}{(\mathrm{e}^u - 1)^2}$$

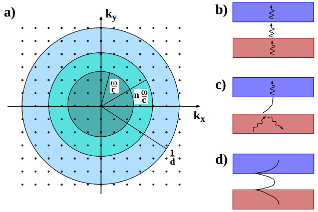
Biehs, Rousseau, Greffet, PRL **105**, 234301 (2010)

Mean transmission coefficient



$$\Phi = \frac{\pi^2 k_B^2 T}{3h} \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_j \Delta T$$





SAB, Rousseau, Greffet, PRL **105**, 234301 (2010)

fundamental limits:

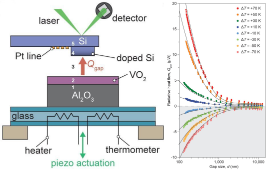
Ben-Abdallah and Joulain, PRB **82**, 121419 (R)(2010)

SAB, Tschikin, Ben-Abdallah, PRL **109**, 104301 (2012)

Miller, Johnson, Rodriguez, PRL **115**, 204302 (2015)

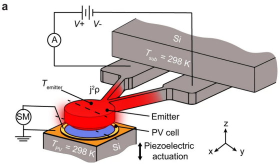
Application of near-field enhanced heat transfer

Thermal management: Diode & Transistor



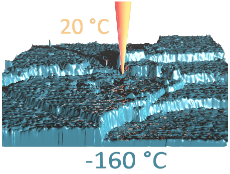
Fiorino et al., ACS Nano **12**, 5774 (2018)
Ben-Abdallah, SAB, PRL **112**, 044301 (2014)

NF thermo-photovoltaics



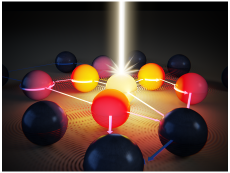
Mittapally et al., Nat. Comm. **12**, 4364 (2021)

NF thermal microscope



Kloppstech et al., Nature Comm. **8**, 14475 (2017)

Many-body effects



SAB, Messina, et al., RMP **93**, 025009 (2021)

Summary

- New laws of thermal radiation in the near-field regime
- Fluctuational electrodynamics as theoretical framework
- Heat flux can surpass blackbody limit due to evanescent waves
- Experimental evidence down to a few nanometer distance
- New heat flux channel in the extreme near-field?